ASTR 7500: Solar & Stellar Magnetism

Hale CGEG Solar & Space Physics

Mark Miesch, Prof. Juri Toomre + HAO/NSO colleagues

Lecture 5 Thurs 5 Feb 2013

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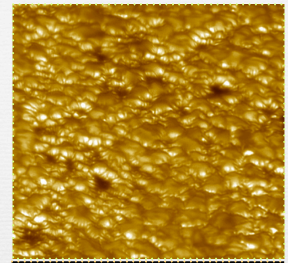
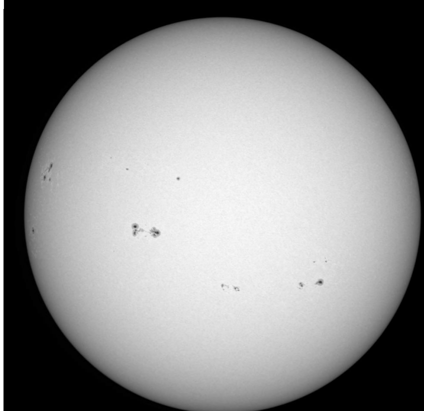
Where did we leave off?

Questions???

The many faces (and scales) of solar convection

$L \sim$ 1-2 Mm, $U \sim$ 1 km s^{-1}, t ~ 10-15 min	***Granulation***
$L \sim$ 5 Mm, $U \sim$ 300 m s^{-1}, t ~ 3-4 hrs	***Mesogranulation***
$L \sim$ 30-35 Mm, $U \sim$ 400 m s^{-1}, t ~ 20 hours	***Supergranulation***
$L \sim$ 100 Mm, $U \sim$ 100 m s^{-1}, t ~ days - months	***Giant Cells***





Of course, the fact that the spots are distributed mainly in two belts parallel to the solar equator, involves no difficulty, for it is easy to conceive how, in more than one way, the Sun's rotation might lead to such a result: but peculiarities permanently attaching to individual points on the solar surface necessarily imply rigid connections, such as are inconsistent with the theory of a gaseous or even of a fluid nucleus.

- Charles A. Young, *The Sun* (1895)

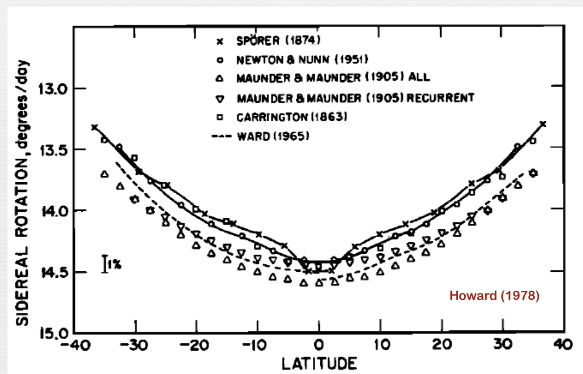
Seeing Spots

Differential Rotation

Sunspots closer to the equator rotate faster

Persistent

The rotation rate determined from spots has not changed by more than a few % since Carrington's measurements spanning 1853-1861 (published in 1863)



Howard (1978)

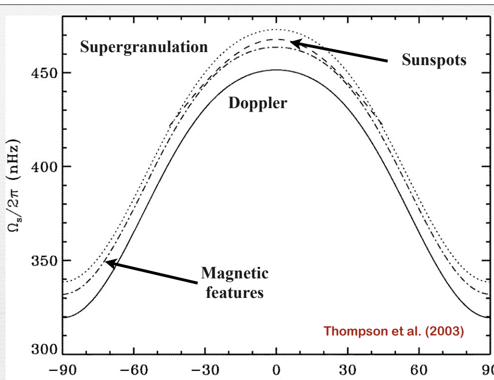
Solar surface rotation

Magnetic features rotate faster than the photospheric plasma!

Interpreted as evidence for a subsurface speed-up in angular velocity

Spots are anchored deeper

Foukal (1972)



Thompson et al. (2003)

Supergranulation moves fastest of all!

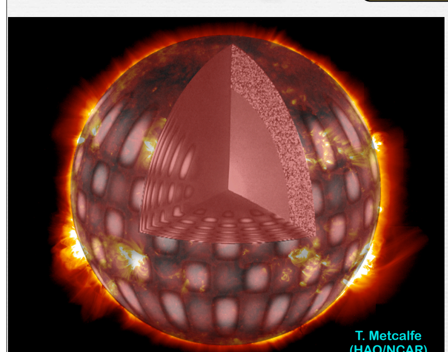
Traveling convective waves?

Gizon, Duvall & Schou (2003)

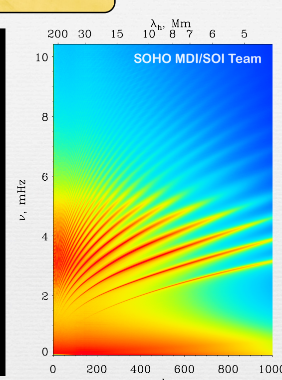
$$\frac{\Omega}{2\pi} = a + b\cos^2\theta + c\cos^4\theta$$

Enter Helioseismology

Peering inside a star



T. Metcalfe (HAO/NCAR)

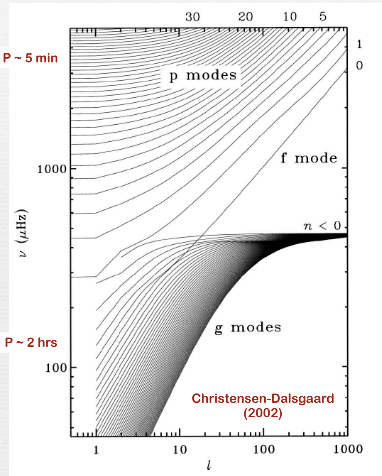


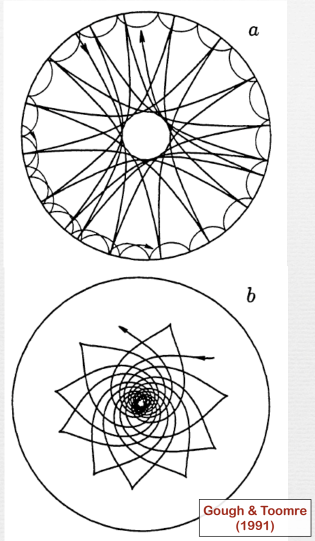
The most reliable observable is Doppler velocity of the photosphere, although intensity may also be used

Global, local oscillations excited by convection

5 min ~ 3.33 mHz

Global Oscillation Modes





Global Rotational Inversions

$$\omega_{nlm} = \omega_{nl0} + m \int_0^R \int_0^\pi K_{nlm}(r,\theta)\Omega(r,\theta) r dr d\theta$$

ω_{nlm} Observed

$$\Delta_{nlm} \equiv \frac{\omega_{nlm} - \omega_{nl0}}{m}$$

Rotational Splitting

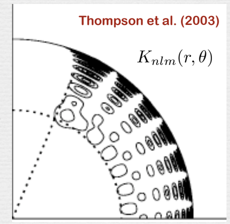
ω_{nl0} , $K_{nlm}(r,\theta)$ Solar Structure Model

$$\sum_{nlm} c_{nlm}(r_0,\theta_0)\Delta_{nlm} = \int_0^R \int_0^\pi \mathcal{K}(r_0,\theta_0;r,\theta)\Omega(r,\theta) r dr d\theta$$

$$= \overline{\Omega}(r_0,\theta_0)$$

$$\mathcal{K}(r_0,\theta_0;r,\theta) = \sum_{nlm} c_{nlm}(r_0,\theta_0) K_{nlm}(r,\theta)$$

$c_{nlm}(r_0,\theta_0)$ **You pick!**



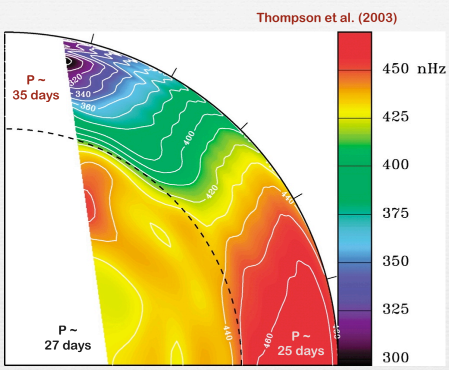
The Internal Rotation of the Sun

Differential Rotation (DR) Monotonic decrease in W of ~ 30% from equator to high latitudes in CZ

Nearly uniform rotation in radiative interior

Convection Implicated as source of DR

Interior rate intermediate relative to CZ



Conical isosurfaces at mid-latitudes

Near-surface shear layer ($0.95R < r < R$)

Tachocline ($0.69R < r < 0.72R$; CZ base = $0.713R$ +/- 0.003)

- ***Toroidal field generation by rotational shear (critical for global dynamo)***
- ***Penetrative convection, internal gravity waves***
- ***Instabilities (magnetic buoyancy, magneto-shear)***
- ***Confinement***

See "The Solar Tachocline", ed. D.W. Hughes, R. Rosner, N.O. Weiss, Cambridge Univ. Press (2007)

Temporal variations

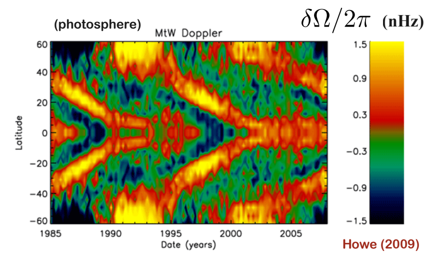
Torsional Oscillations (P ~ 22 yr)

~ 1% peak-to-peak

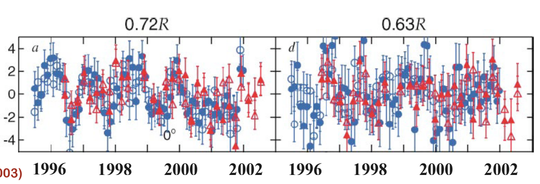
correlated with magnetic activity bands

polar, equatorial branches

see Rempel (2007) for a nice investigation of what might cause these



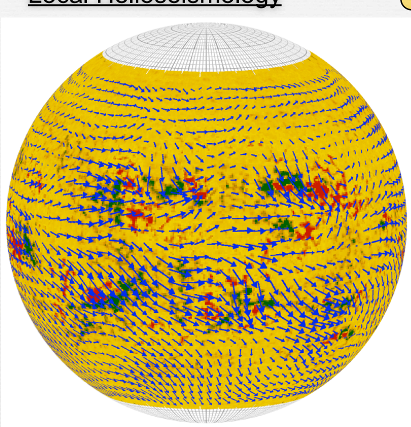
Tachocline Oscillations? (P ~ 1.3 yr)

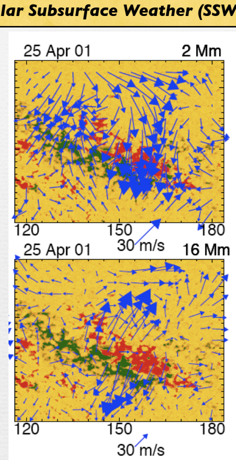


Thompson et al. (2003)

Local Helioseismology

Solar Subsurface Weather (SSW)



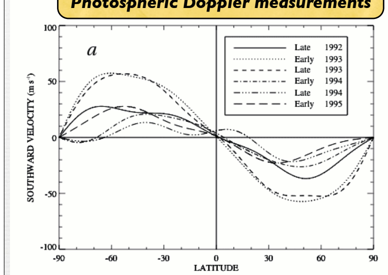


B. Hindman, D. Haber, J. Toomre (JILA/Univ. of Colorado)

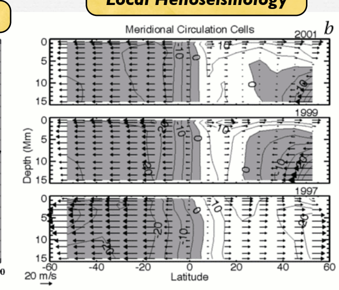
Inferring subsurface flows from local high-wavenumber, non-resonant acoustic wave fields (see Gizon & Birch http://solarphysics.livingreviews.org)

Meridional Flow

Photospheric Doppler measurements



Local Helioseismology



Poleward near surface ($r > 0.97R$) at latitudes < 60° (unknown elsewhere)

Amplitude ~ 10-20 m s^{-1} but highly variable

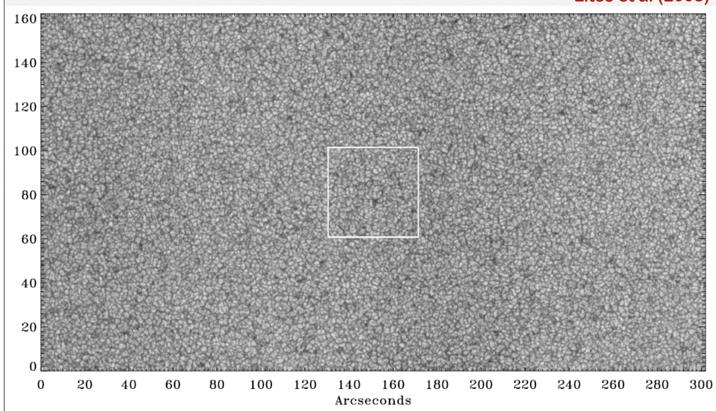
Possible evidence for multiple cells at high latitudes, deeper levels

Solar cycle variations; convergence into activity bands (near surface)

Where do these mean flows come from?

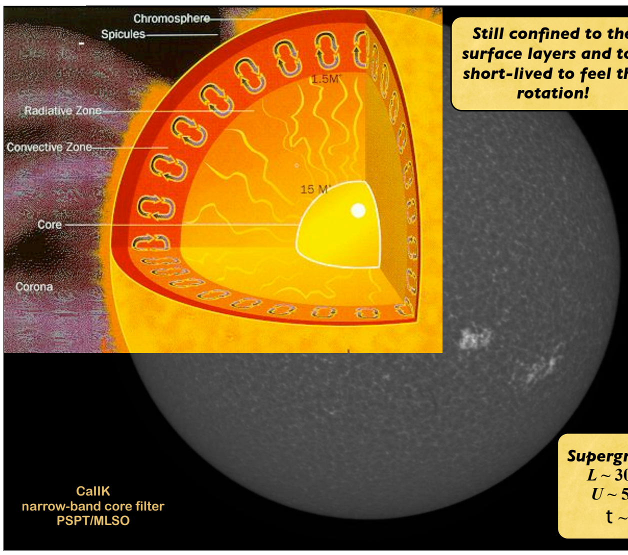
Solar Convection: Granulation

Lites et al (2008)



L ~ 1-2 Mm
U ~ 1 km s^{-1}
t ~ 10-15 min

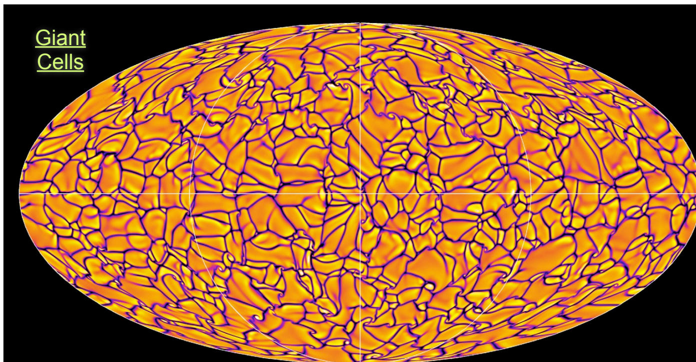
Solar Dermitology
This is just the thermal boundary layer!

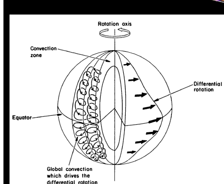


Still confined to the surface layers and too short-lived to feel the rotation!

Supergranulation
L ~ 30-35 Mm
U ~ 500 m s^{-1}
t ~ 20 hr

Giant Cells





Big enough and slow enough to feel the spherical geometry and rotation

L ~ 100 Mm
U ~ 100 m s^{-1}
t ~ days - months

How do mean flows arise from convection?

conservation of momentum in MHD (inertial reference frame)

$$\rho\frac{\partial \mathbf{v}}{\partial t} + \rho\left(\mathbf{v}\cdot\boldsymbol{\nabla}\right)\mathbf{v} = -\boldsymbol{\nabla}P + \rho\mathbf{g} + \frac{1}{4\pi}\left(\boldsymbol{\nabla}\times\mathbf{B}\right)\times\mathbf{B} + \boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{D}}$$

Also conservation of mass

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla}\cdot\left(\rho\mathbf{v}\right)$$

Also -
ideal gas equation of state,
Induction equation, $\nabla\cdot B = 0$

And conservation of energy

$$\rho T\left(\frac{\partial S}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}S\right) = -\boldsymbol{\nabla}\cdot\mathbf{q} + \Phi_v + \Phi_B$$

Zonal (L) component of the momentum equation

multiply by and average over longitude

$$\frac{\partial}{\partial t}\left(\rho\mathcal{L}\right) + \left\langle\rho\mathbf{v}_m\right\rangle\cdot\boldsymbol{\nabla}\mathcal{L} = \mathcal{F}$$

No assumptions beyond basic MHD!

$$\mathcal{F} = -\boldsymbol{\nabla}\cdot\left[\lambda\left\langle\rho\mathbf{v}'v_\phi'\right\rangle - \lambda\left\langle\mathbf{B}B_\phi\right\rangle - \rho\nu\lambda^2\boldsymbol{\nabla}\Omega\right]$$

Reynolds stress ***Lorentz force*** ***Viscous diffusion***

(Maxwell stress + mean magnetic tension)

Conservation of angular momentum

moment arm $\lambda = r\sin\theta$

specific angular momentum $\mathcal{L} = \lambda^2\Omega = \lambda\left\langle v_\phi\right\rangle$

Meridional (r,θ) components of the momentum equation

divide the momentum equation by ρ, average over longitude, take the curl, and grab the ϕ component of that

$$\frac{\partial \langle \omega_\phi \rangle}{\partial t} = \lambda \frac{\partial \Omega^2}{\partial z} + $$

Fine...So what do these *mean*?

ion of omentum

baroclinic term $\mathcal{B} = $

Reynolds ... *usion*

$$\mathcal{G} = \left\{ \nabla \times \left\langle (\nabla \times \mathbf{v}') \times \mathbf{v}' + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho} + \frac{\nabla \cdot \mathcal{D}}{\rho} \right\rangle \right\} \cdot \hat{\phi}$$

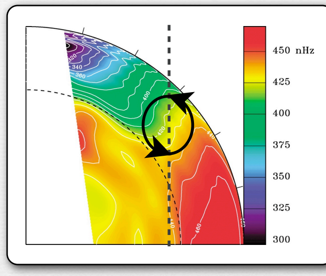
$$+ \{ \nabla \times [\nabla \times \langle \mathbf{v}_m \rangle] \times \langle \mathbf{v}_m \rangle \} \cdot \hat{\phi} + \left\langle \frac{\nabla P \times \nabla \rho}{\rho^2} \right\rangle - \mathcal{B}$$

meridional flow self-advection — *thermodynamic fluctuations*

What do these equations *mean*?

Start with the meridional one

$$\frac{\partial \langle \omega_\phi \rangle}{\partial t} = \lambda \frac{\partial \Omega^2}{\partial z} + \mathcal{B} + \mathcal{G}$$



We know what the first term must be doing from helioseismology

Now interpret <> as a time average as well as a longitudinal average
What would it take to establish a steady state?

Something must be trying to establish a CW circulation in the NH, to oppose the Coriolis-induced CCW flow

***Most current models** (theoretical arguments, mean-field models, convection simulations) **point to the baroclinic term** B*

What do these equations *mean (cont.)*?

if the stratification is nearly adiabatic and hydrostatic (which it is in the CZ), then

$$\mathcal{B} \approx -\frac{g}{rC_P} \frac{\partial \langle S \rangle}{\partial \theta}$$

if G is indeed negligible in the deep CZ then we get

$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r\lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}$$

Thermal Wind Balance

Axial differential rotation is linked directly to the latitudinal entropy gradient

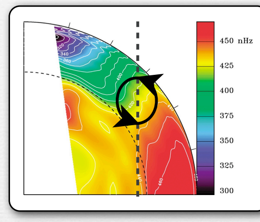
We know the LHS from helioseismology so - assuming this balance is satisfied - that tells us the RHS

What does it tell us?
Warm poles ($\partial\langle S\rangle/\partial\theta < 0$ in NH) needed to offset inertia of differential rotation

What do these equations *mean (cont.)*?

Now go back to the time dependent equation

$$\frac{\partial \langle \omega_\phi \rangle}{\partial t} = \lambda \frac{\partial \Omega^2}{\partial z} - \frac{g}{rC_P} \frac{\partial \langle S \rangle}{\partial \theta}$$



Inertia of the DR induces CCW circulation

Baroclinicity induces CW circulation

In a steady state they balance but one must be the "driver" and one the response

In other words, one must accelerate the MC and the other must resist it until a balance is reached

The observed poleward flow in solar surface layers (CCW) suggests that the DR establishes the MC through the Coriolis/centrifugal force while baroclinicity regulates the * profile

What do these equations *mean*?

So - to summarize - the meridional components of the momentum equation regulate the differential rotation profile through thermal wind balance (TWB)

$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r\lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}$$

However - note two very important caveats

Caveat 1: This doesn't tell us why the equator spins faster than the poles

***For a given S, you can take any solution * and add an arbitrary cylindrical profile *'( ) and it will still be a solution** (geostrophic degeneracy)*
So, TWB is consistent with an infinite number of * profiles, some solar-like (fast equator, slow poles), some anti-solar (slow equator, fast poles)

Caveat 2: This doesn't tell us what the steady-state MC profile will be

In a statistically steady state, the MC falls out of this equation

What do these equations *mean*?

First address caveat 1 from the previous slide:
Why does the equator spin faster than the poles?

Recall our mean zonal momentum equation

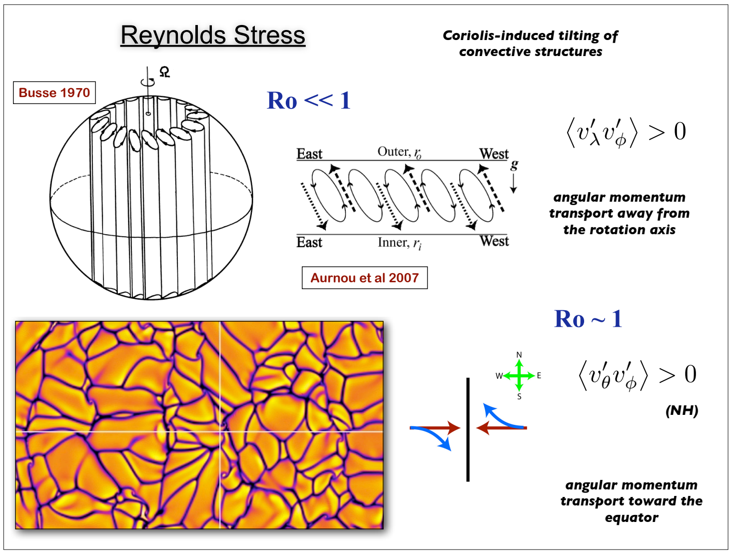
$$\frac{\partial}{\partial t}(\rho \mathcal{L}) + \langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$$

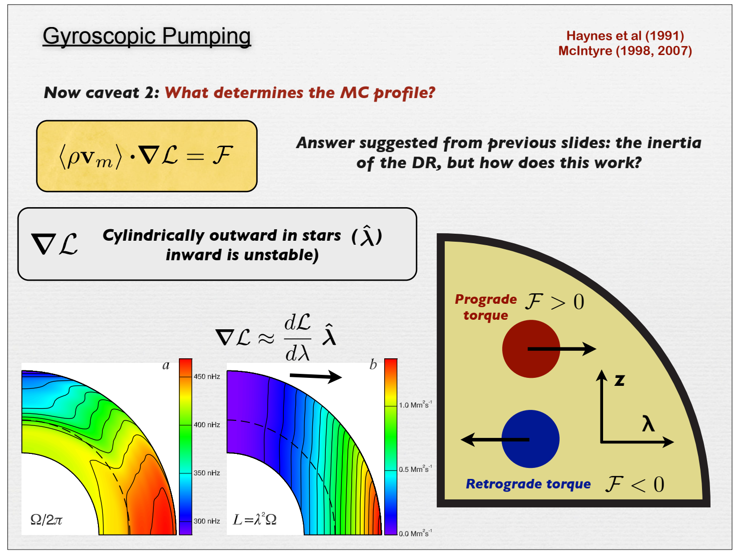
Steady state

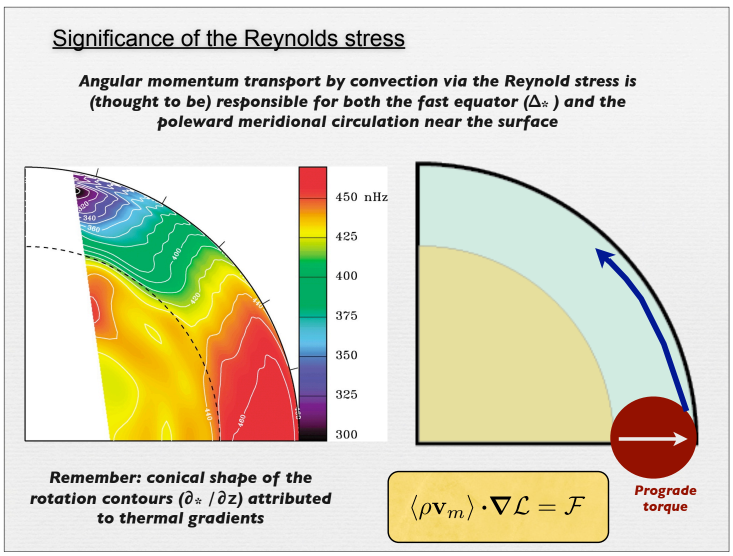
$$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F} \qquad \mathcal{F} \approx -\nabla \cdot \left[\lambda \langle \mathbf{v}' v_\phi' \rangle \right]$$

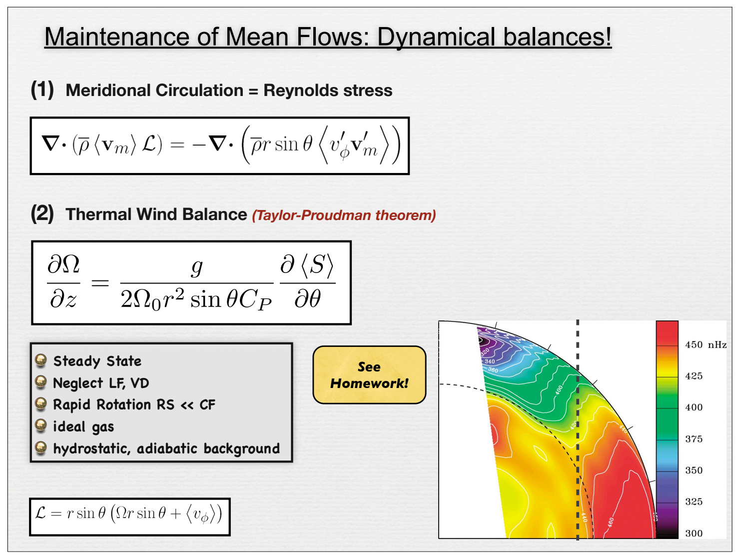
Reynolds stress!

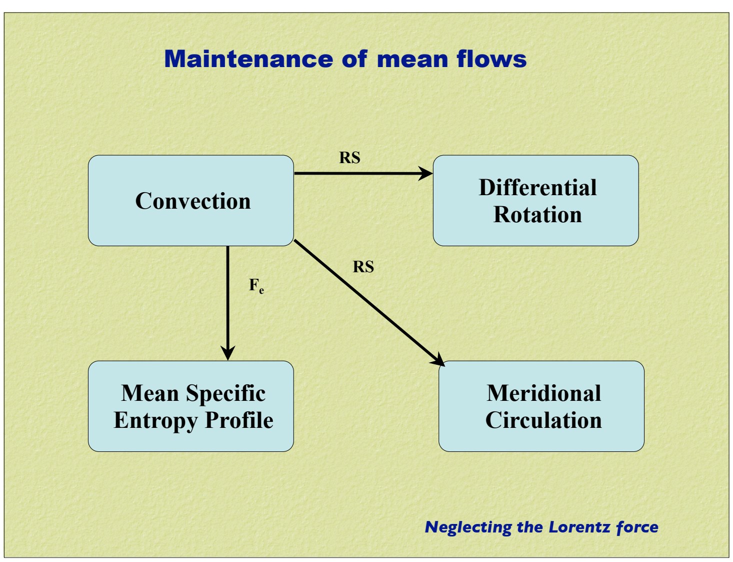
Angular momentum transport by convection via the Reynolds stress must establish Δ*

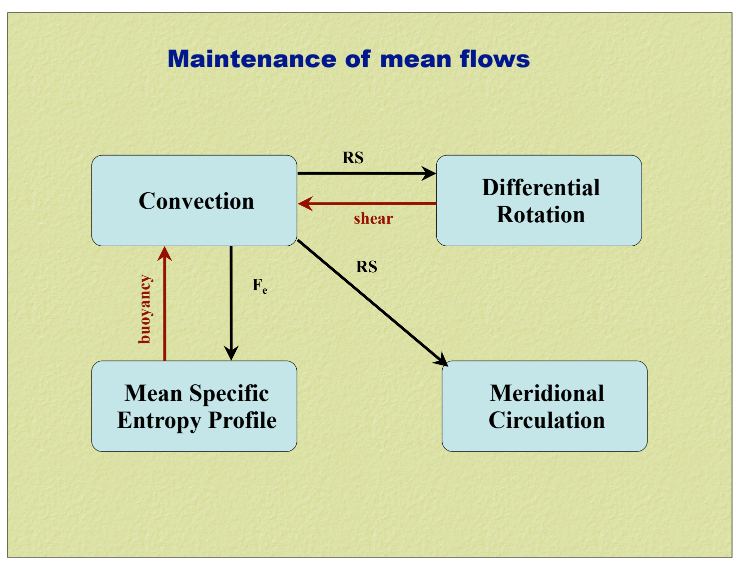

Reynolds Stress
Coriolis-induced tilting of convective structures
Busse 1970
Ω
Ro << 1
East
Outer, r_o
West
g
East
Inner, r_i
West
Aurnou et al 2007
$\langle v'_\lambda v'_\phi \rangle > 0$
angular momentum transport away from the rotation axis
Ro ~ 1
N
W
E
S
$\langle v'_\theta v'_\phi \rangle > 0$
(NH)
angular momentum transport toward the equator


Gyroscopic Pumping
Haynes et al (1991)
McIntyre (1998, 2007)
Now caveat 2: What determines the MC profile?
$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$
Answer suggested from previous slides: the inertia of the DR, but how does this work?
$\nabla \mathcal{L}$
Cylindrically outward in stars $(\hat{\lambda})$
inward is unstable)
$\nabla \mathcal{L} \approx \frac{d\mathcal{L}}{d\lambda} \hat{\lambda}$
a
b
Ω/2π
L=λ²Ω
Prograde torque
$\mathcal{F} > 0$
z
λ
Retrograde torque
$\mathcal{F} < 0$

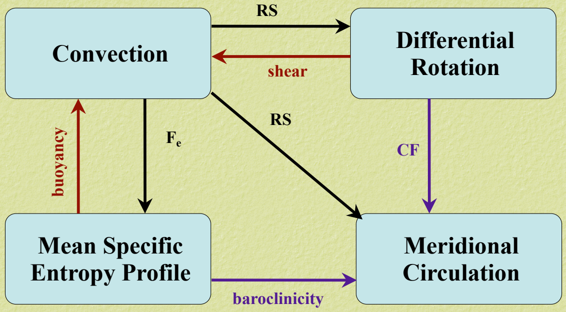

Significance of the Reynolds stress
Angular momentum transport by convection via the Reynold stress is (thought to be) responsible for both the fast equator (Δ*) and the poleward meridional circulation near the surface
450 nHz
425
400
375
350
325
300
Remember: conical shape of the rotation contours (∂* / ∂z) attributed to thermal gradients
$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$
Prograde torque


Maintenance of Mean Flows: Dynamical balances!
(1) Meridional Circulation = Reynolds stress
$\nabla \cdot (\bar{\rho} \langle \mathbf{v}_m \rangle \mathcal{L}) = -\nabla \cdot \left(\bar{\rho} r \sin\theta \left\langle v'_\phi \mathbf{v}'_m \right\rangle \right)$
(2) Thermal Wind Balance (Taylor-Proudman theorem)
$\frac{\partial \Omega}{\partial z} = \frac{g}{2\Omega_0 r^2 \sin\theta C_P} \frac{\partial \langle S \rangle}{\partial \theta}$
Steady State
Neglect LF, VD
Rapid Rotation RS << CF
ideal gas
hydrostatic, adiabatic background
See Homework!
$\mathcal{L} = r \sin\theta \left(\Omega r \sin\theta + \langle v_\phi \rangle \right)$
450 nHz
425
400
375
350
325
300

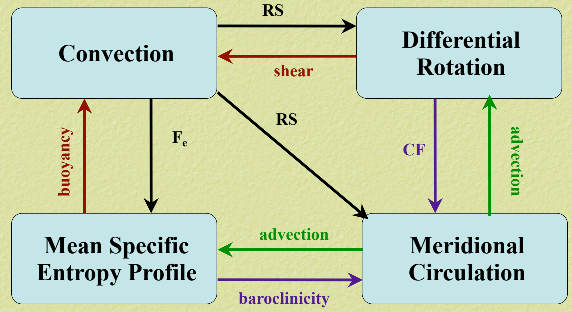

Maintenance of mean flows
Convection
RS
Differential Rotation
F_e
RS
Mean Specific Entropy Profile
Meridional Circulation
Neglecting the Lorentz force


Maintenance of mean flows
Convection
RS
shear
Differential Rotation
buoyancy
F_e
RS
Mean Specific Entropy Profile
Meridional Circulation

Maintenance of mean flows

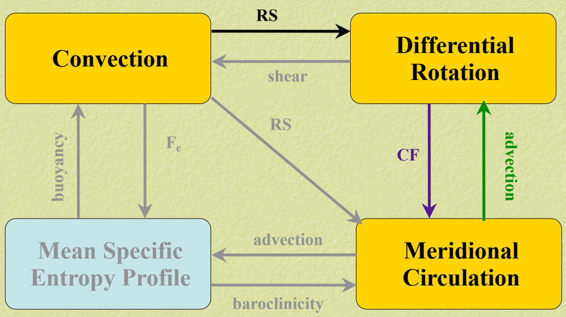


Maintenance of mean flows



Maintenance of mean flows

(1) Angular momentum balance

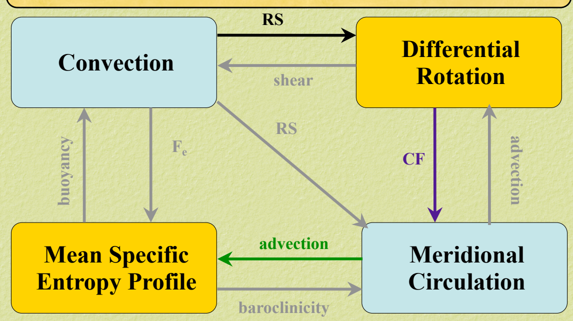


$$\nabla\cdot\left(\bar{\rho}\langle\mathbf{v}_m\rangle\mathcal{L}\right) = -\nabla\cdot\left(\bar{\rho} r\sin\theta\left\langle v'_\phi \mathbf{v}'_m\right\rangle\right)$$

Maintenance of mean flows

(2) thermal wind balance via mechanical forcing

Only works for stable (subadiabatic) stratification!

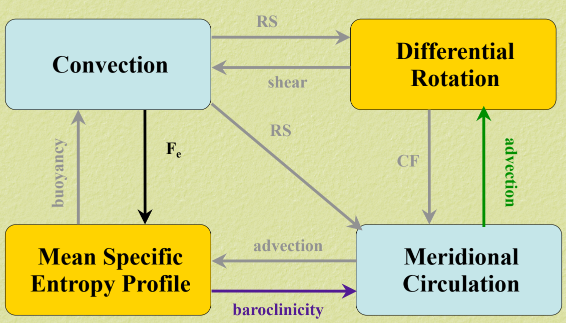


$$\frac{\partial\Omega}{\partial z} = \frac{g}{2\Omega_0 r^2\sin\theta C_P}\frac{\partial\langle S\rangle}{\partial\theta}$$

$$\frac{\partial\langle S\rangle}{\partial t} = -\langle v_r\rangle\frac{\partial\langle S\rangle}{\partial r} + \dots$$

Maintenance of mean flows

(2) thermal wind balance via thermal forcing

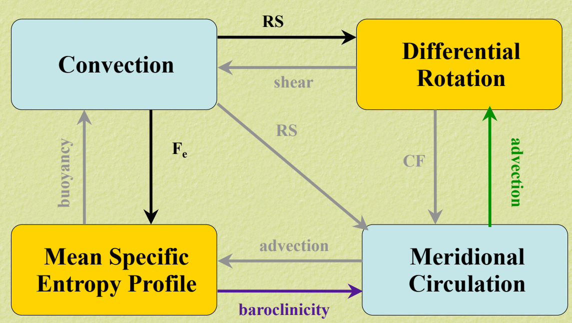


$$\frac{\partial\Omega}{\partial z} = \frac{g}{2\Omega_0 r^2\sin\theta C_P}\frac{\partial\langle S\rangle}{\partial\theta}$$

$$\nabla\cdot\left(\bar{\rho}\langle\mathbf{v}_m\rangle\mathcal{L}\right) = \bar{\rho}\langle\mathbf{v_m}\rangle\cdot\nabla\mathcal{L} = 0$$

Maintenance of mean flows

(2) thermal wind balance via thermal forcing

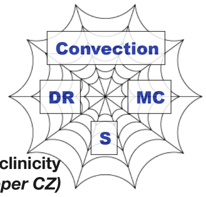


$$\frac{\partial\Omega}{\partial z} = \frac{g}{2\Omega_0 r^2\sin\theta C_P}\frac{\partial\langle S\rangle}{\partial\theta}$$

Only works if there's a RS contribution as well!

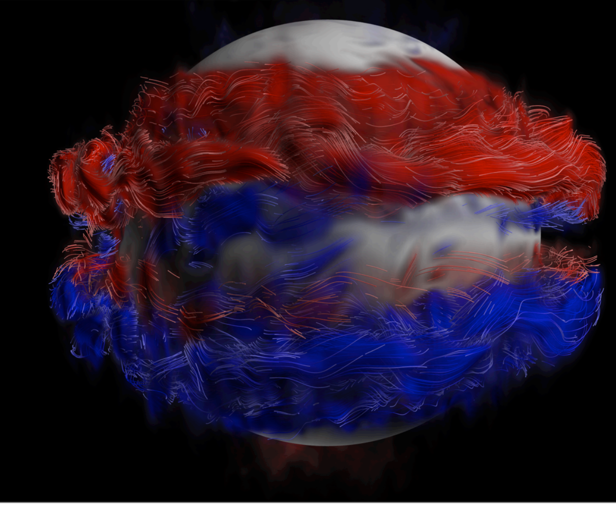
Summary: The Solar Internal Rotation

- **Differential Rotation**
  - Photospheric measurements
  - Global, local helioseismology
  - Monotonic decrease from equator to pole
  - Conical mid-latitude contours
  - Tachocline, near-surface shear layer
  - Maintained by convective Reynolds stress, baroclinicity
  - Thermal wind balance in lower CZ *(but not in upper CZ)*

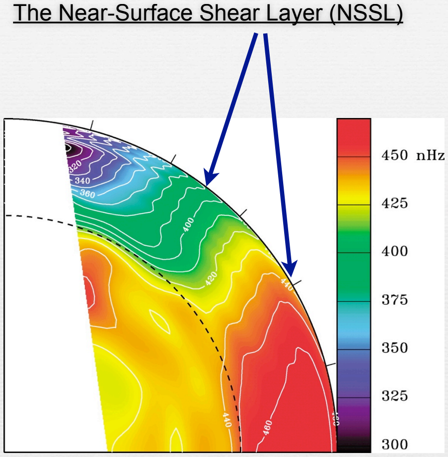


- **Meridional Circulation**
  - Photospheric measurements, local helioseismology
  - Poleward (but variable) in surface layers
  - unknown deeper down
  - maintained mainly by inertia of differential rotation

Next Lecture: Convective Dynamos!



The Near-Surface Shear Layer (NSSL)



A nice illustration of gyroscopic pumping

A simple thought experiment

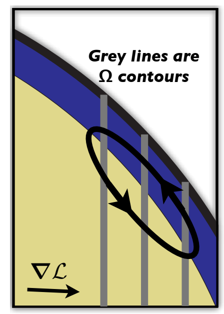
Consider a flow that is axisymmetric, non-magnetic, non-diffusive, and that the stratification is adiabatic (∇S=0)

Then $\mathcal{B} = \mathcal{G} = 0$

And our zonal vorticity equation is just...

$$\frac{\partial}{\partial t}\langle\omega_\phi\rangle = \lambda\frac{\partial\Omega^2}{\partial z}$$

A steady state is provided by the Taylor-Proudman Theorem $\Omega = \Omega(\lambda)$



Now, at some time t_0, start applying a retrograde zonal force $\mathcal{F}$ in the NSSL: The system adjusts to a new equilibrium state, still with a cylindrical rotation

...provided $\int \mathcal{F}\, dV = 0$

Note: the mere existence of a retrograde zonal torque does not guarantee the presence of a NSSL...but it does establish a meridional circulation

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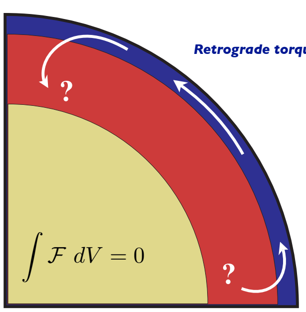
Illustration: Gyroscopic Pumping in the solar NSSL

The same retrograde torque that gives rise to the slower Ω also gives rise to the poleward meridional flow



$$\langle\rho\mathbf{v}_m\rangle\cdot\nabla\mathcal{L} = \mathcal{F}$$

$$\langle\rho v_\lambda\rangle \approx \frac{\mathcal{F}}{d\mathcal{L}/d\lambda} \approx \frac{\mathcal{F}}{2\lambda\Omega_0}$$

simplest case Ro << 1

The net longitudinal force determines the meridional flow

(Is poleward meridional flow a surface effect? cf. Hathaway 2011)

$$\langle\rho v_\lambda\rangle = \frac{\partial\Psi}{\partial z} \qquad \langle\rho v_z\rangle = -\frac{1}{\lambda}\frac{\partial}{\partial\lambda}(\lambda\Psi)$$

$$\Psi(\lambda, z) = \left(\frac{d\mathcal{L}}{d\lambda}\right)^{-1}\int_{z_b}^{z}\mathcal{F}(\lambda, z')dz'$$

$$z_b = \left(R^2 - \lambda^2\right)^{1/2}$$