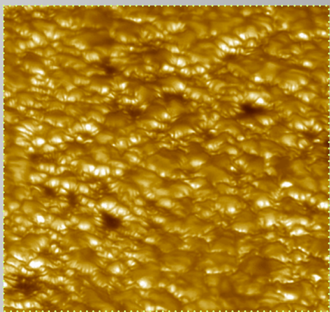
ASTR 7500: Solar & Stellar Magnetism

Hale CGEG Solar & Space Physics



Mark Miesch, Prof. Juri Toomre + HAO/NSO colleagues

Lecture 3 Tues 29 Jan 2013

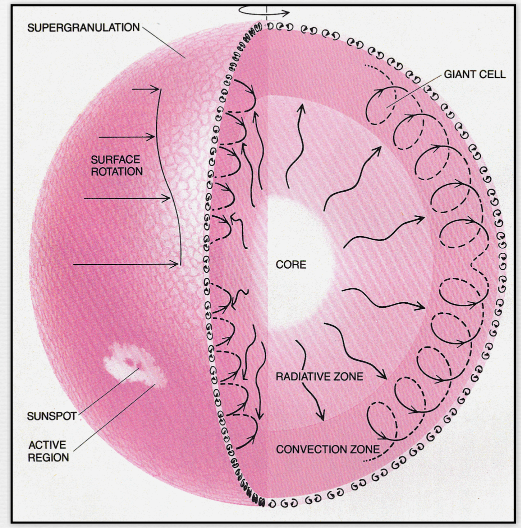
zeus.colorado.edu/astr7500-toomre

The Solar Interior

Where does this picture come from?

What do we really know about it?

What don't we know?



The Basic (Macroscopic) Equations of Solar (and Stellar) Structure

e.g. Hansen & Kawaler (1994)

Consider a sphere of fluid (gas or plasma)

(see lectures 7-8)

...subject to the equations of Magnetohydrodynamics (MHD)

...Now, as a first approximation, ignore the "M" and the "D" *(for the most part)*

Conservation of momentum yields hydrostatic balance

$$\nabla P = -\rho \nabla \Phi$$

Newton's law of gravity (via Gauss & Poisson) yields

$$\nabla^2 \Phi = 4\pi G \rho$$

Conservation of energy yields

$$\nabla \cdot (\mathbf{F}_r + \mathbf{F}_c) = \epsilon$$

Radiative heat flux | Convective heat flux | Energy generation rate (erg cm^{-3})

integral form of this may be more intuitive (thanks again to Mr. Gauss)

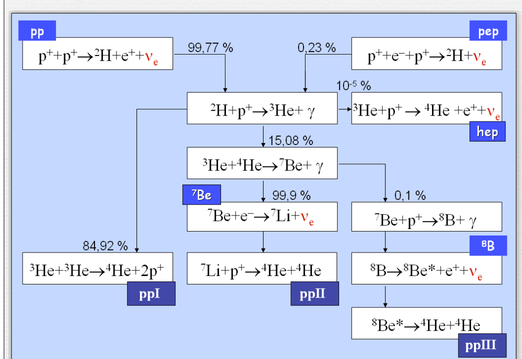
$$\int (\mathbf{F}_r + \mathbf{F}_c) \cdot d\mathbf{S} = \int \epsilon \, dV$$

The (Microscopic) Nitty Gritty

$$\nabla \cdot (\mathbf{F}_r + \mathbf{F}_c) = \epsilon$$

Lots of complex physics hidden in this ostensibly simple equation!

Consider first the energy generation term



The Sun gets most of its energy from this p-p chain

$$\epsilon \propto \rho T^4$$

Fuses 700 million tons *(equivalent to 100 great pyramids of Giza)* ***of protons into He nuclei every second, releasing*** 4×10^{33} erg *(equivalent to 10 billion Hydrogen bombs)*

..Yet the energy released per second per unit mass (L/M) is less than a wood fire

The (Microscopic) Nitty Gritty

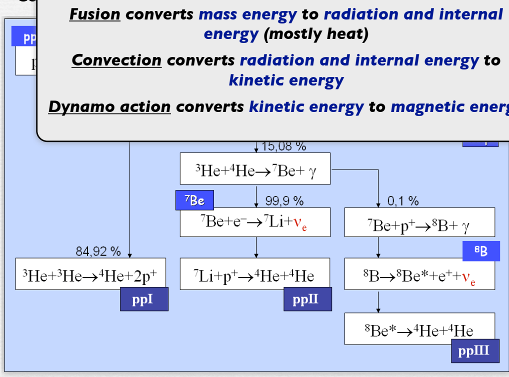
$$\nabla \cdot (\mathbf{F}_r + \mathbf{F}_c) = \epsilon$$

As we will see, this is the ultimate energy source for solar activity!
(the buck stops here for the whole course!)

Fusion converts mass energy to radiation and internal energy (mostly heat)

Convection converts radiation and internal energy to kinetic energy

Dynamo action converts kinetic energy to magnetic energy



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The (Microscopic) Nitty Gritty (cont.)

$$\nabla \cdot (\mathbf{F}_r + \mathbf{F}_c) = \epsilon$$

Now consider the radiative flux $\mathbf{F}_r$

The solar interior is dense enough to radiate essentially as a blackbody

Energy density (erg cm^{-3}) ***of radiation is:*** $E_r = aT^4$

But the temperature will varies with position! *(it's hotter in the core!)*

So, we might expect $\mathbf{F}_r \propto \nabla E_r = 4aT^3 \nabla T$

Indeed, a more careful derivation yields (e.g. Hansen & Kawaler 1994)

$$\mathbf{F}_r = -\frac{acT^3}{3\sigma\rho} \nabla T$$

Where $\sigma\alpha$ ***is the (frequency-averaged) opacity***

The (Microscopic) Nitty Gritty (cont.)

Opacity: Lots of complex physics here too!

Sources of opacity

Electron Scattering
photons bouncing off of free electrons also called Thompson or Compton scattering (low/ high energy)

$$\sigma \approx \frac{\sigma_e n_e}{\rho} \approx 0.2(1+X)$$

(for a fully ionized H/He mix)

Free-Free Absorption
An electron absorbs a photon in the vicinity of an ion (inverse Bremsstrahlung)

Bound-free Absorption
An electron bound in an ion/atom absorbs a photon and is jettisoned (ionization)

Bound-Bound Opacity
An electron bound in an ion/atom absorbs a photon and bounces to a higher quantum energy state

Kramers opacity

$$\sigma \propto \rho T^{-3.5}$$

H^- Opacity
Extra electron in a Hydrogen atom gets knocked off by a passing photon

$$\sigma \propto \rho^{1/2} T^9$$

More...

The (Microscopic) Nitty Gritty (cont.)

Equation of state (EOS): Lots of complex physics here too!

F_r, involve the temperature (so does F_c, as we'll see), so we also need an ***Equation of state***

$$P = P(\rho, T, \mathcal{C})$$

..or, equivalently, $T = T(\rho, P, \mathcal{C})$

Simplest case is an ideal, monatomic gas - good approximation in most of the solar interior except near the surface

$$P = nkT = \frac{k}{\mu}\rho T = \mathcal{R}\rho T$$

$$\mathcal{R} = C_P - C_v = C_v(\gamma - 1)$$

More sophisticated and exotic EOS needed for:

*Solar and stellar **surface layers** (and cool, low-mass stars) where atomic and molecular species recombine (partially ionized)*

***High-mass stars** where copious amounts of radiation contribute to internal energy & pressure*

***Compact objects** like white dwarfs and neutron stars where degeneracy (& even crystallization) becomes important*

Recap: Where are we going with this?

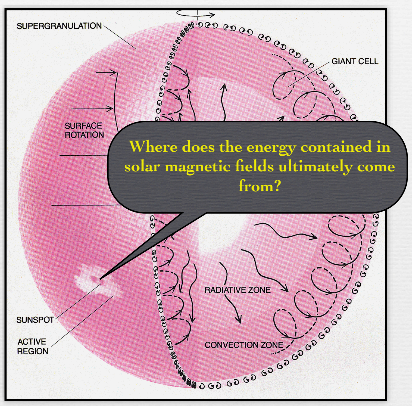
$$\nabla P = -\rho \nabla \Phi$$

$$\nabla^2 \Phi = 4\pi G \rho$$

$$\nabla \cdot (\mathbf{F}_r + \mathbf{F}_c) = \epsilon$$

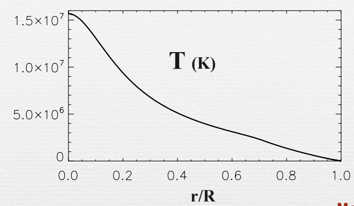
Still haven't talked about this yet but we're getting there!

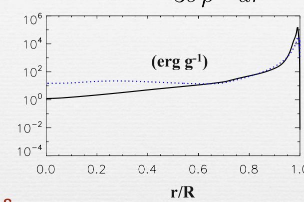
*(Note - discussion so far is **spherically symmetric** so replace ∇ operators with **radial components**)*



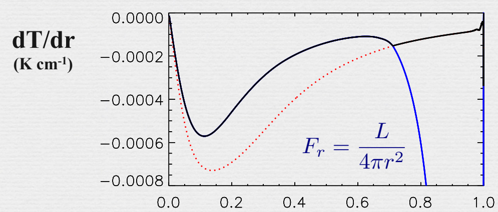
A Solar Model: Sample Results

$$F_r = -\frac{acT^3}{3\sigma\rho}\frac{dT}{dr}$$





Model S (Christensen-Dalsgaard et al 1996)



The temperature gradient isn't big enough in the solar envelope to carry out the solar luminosity!

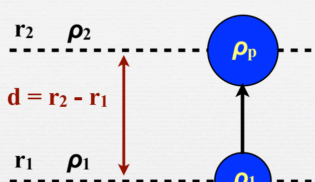
Why?

Convection!

Consider a "parcel" of fluid at an initial radius r_1 where the ambient density is ρ_1

Now move to a (slightly) higher radius r_2 where the ambient density is $\rho_2 < \rho_1$

From the second law of thermodynamics for a reversible process with no heating or cooling, then the entropy S of the parcel will not change



$$S = C_P \ln \frac{P^{1/\gamma}}{\rho}$$

$$\rho_p = \rho_1 \left(\frac{P_p}{P_1}\right)^{1/\gamma} = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} = \rho_1 \left(1 + \frac{d}{P_1}\frac{dP}{dr}\right)^{1/\gamma} \approx \rho_1 + d\,\frac{\rho}{\gamma P}\frac{dP}{dr}$$

$$\rho_2 = \rho_1 + d\,\frac{d\rho}{dr}$$

$\rho_p < \rho_2$ **if**

$$\frac{1}{\gamma}\frac{d\ln P}{dr} - \frac{d\ln\rho}{dr} = \left(\frac{1}{\gamma} - 1\right)\frac{d\ln P}{dr} - \frac{d\ln T}{dr} < 0$$

using the hydrostatic balance and ideal gas expressions on previous slides yields

$$\frac{dT}{dr} < -\frac{g}{C_P}$$

Work this out yourself!

Convection!

Consider a ... radius r_1 w...

Now move ... where th...

From the ... for a rever... cooling, t...

Called the Schwarzschild criterion

Equivalent to

$$\frac{dS}{dr} < 0$$

Necessary but not sufficient for the onset of convection

Thermal & viscous diffusion can kill it thus the concept of the *critical Rayleigh number* (see lab exercise)

Efficient convection mixes entropy

$$\frac{dS}{dr} \rightarrow 0$$

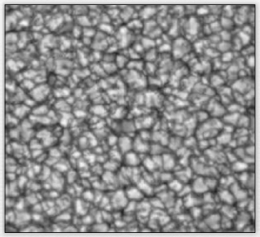
So temperature gradient is never much steeper than adiabatic (except near the surface)

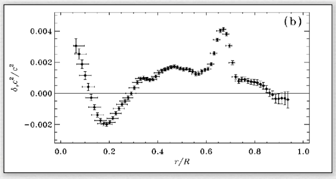
using the hydrostatic balance and ideal gas expressions on previous slides yields

$$\frac{dT}{dr} < -\frac{g}{C_P}$$

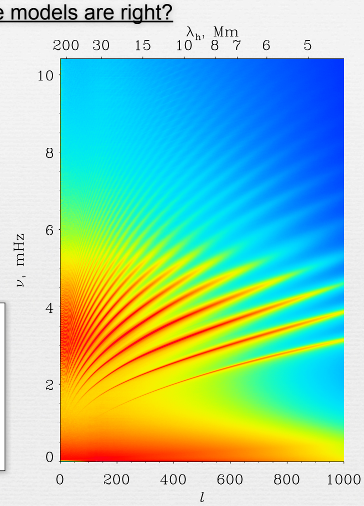
Work this out yourself!

How do we know these structure models are right?





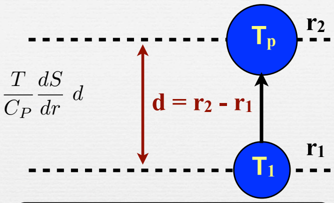
Christensen-Dalsgaard (2002)



Mixing Length Theory

Consider our parcel again

$$\Delta T = T_p - T_2 = \left(-\frac{g}{C_P} - \frac{dT}{dr}\right) d = -\frac{T}{C_P}\frac{dS}{dr} d$$



Then say that after it travels a distance L (called the mixing length) it dissolves, depositing its excess thermal energy

$$\Delta E = \rho C_P \Delta T = -\rho T L \frac{dS}{dr}$$

The associated energy flux is then

$$F_c = w\Delta E = -w\rho T L \frac{dS}{dr}$$

The vertical velocity is (roughly) $w = \Sigma L$

Which gives

$$F_c = \frac{\rho T C_P \Sigma^3 L^2}{g}$$

$$\frac{\partial \Delta T}{\partial t} = -\frac{T}{C_P}\frac{dS}{dr} w$$

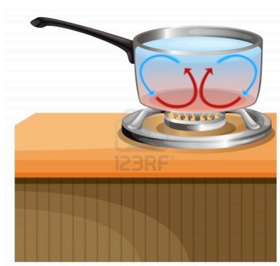
$$\frac{\partial w}{\partial t} = -\frac{\Delta\rho}{\rho} g \approx \frac{\Delta T}{T} g$$

$$\frac{\partial^2 w}{\partial t^2} = \Sigma^2 w = -\frac{g}{C_P}\frac{dS}{dr} w$$

$$\Sigma = \left(-\frac{g}{C_P}\frac{dS}{dr}\right)^{1/2}$$

(real) Convection

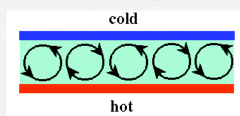
- **Fundamental Aspects**
  - Plumes
  - Boundary Layers
  - Rotation, stratification, magnetism
  - Spherical Geometry
- **Application to the Sun and Stars**
  - Granulation
  - Mesogranulation
  - Supergranulation
  - Giant Cells



NCAR

CU, January, 2012

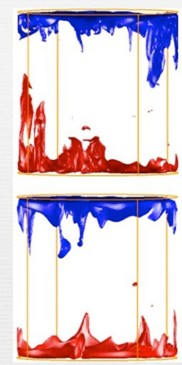
Rayleigh-Bénard Convection



Bénard (1900)
Rayleigh (1916)

Chandrasekhar (1961)

Ahlers, Grossman & Lohse (2009, Rev. Mod. Phys, 81, 503)



Ra = 10^8, Pr = 0.7, 6.4

Zhong et al (2009)

$$\mathrm{Ra} = \frac{\alpha \Delta g D^3}{\nu\kappa} \qquad \mathrm{Pr} = \frac{\nu}{\kappa}$$

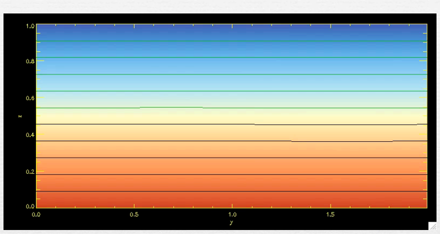
Question: What happens as you decrease ν,κ while keeping everything else the same, including Pr?

Answer: Re, Nu increase

$$\mathrm{Re} = \frac{UD}{\nu} \qquad \mathrm{Nu} = \frac{H}{k\Delta D^{-1}}$$

$$k = C_P \rho\kappa$$

Plumes and Boundary Layers!



What is a boundary layer?

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \kappa\nabla^2 T$$

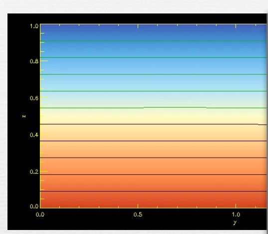
$$\frac{UT}{L} \sim \kappa\frac{T}{\delta_T^2} \qquad \mathrm{Pe} = \frac{UL}{\kappa}$$

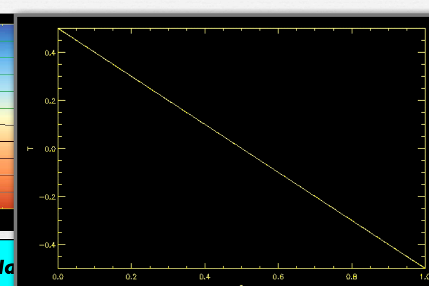
$$\delta_T \sim L\,\mathrm{Pe}^{-1/2}$$

Simulation of 2D convection that you can do yourself!

Done with IDL routines and laboratory exercise that will be distributed to you

Plumes and Boundary Layers!





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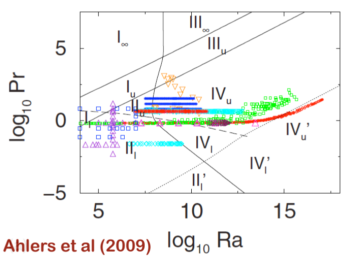
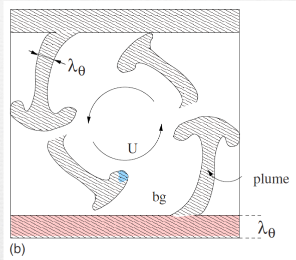
Plumes and *Boundary Layers!*



Ahlers et al (2009)



Regime	Dominance of	BLs	Nu	Re
I_l	$\epsilon_{u,BL}, \epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$Ra^{1/4}Pr^{1/8}$	$Ra^{1/2}Pr^{-3/4}$
I_u		$\lambda_u > \lambda_\theta$	$Ra^{1/4}Pr^{-1/12}$	$Ra^{1/2}Pr^{-5/6}$
I_∞		$\lambda_u = L/4 > \lambda_\theta$	$Ra^{1/5}$	$Ra^{3/5}Pr^{-1}$
II_l	$\epsilon_{u,bulk}, \epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$Ra^{1/5}Pr^{1/5}$	$Ra^{2/5}Pr^{-3/5}$
II_u		$\lambda_u > \lambda_\theta$	$Ra^{1/5}$	$Ra^{2/5}Pr^{-2/3}$
III_u	$\epsilon_{u,BL}, \epsilon_{\theta,bulk}$	$\lambda_u > \lambda_\theta$	$Ra^{3/7}Pr^{-1/7}$	$Ra^{4/7}Pr^{-6/7}$
III_∞		$\lambda_u = L/4 > \lambda_\theta$	$Ra^{1/3}$	$Ra^{2/3}Pr^{-1}$
IV_l	$\epsilon_{u,bulk}, \epsilon_{\theta,bulk}$	$\lambda_u < \lambda_\theta$	$Ra^{1/2}Pr^{1/2}$	$Ra^{1/2}Pr^{-1/2}$
IV_u		$\lambda_u > \lambda_\theta$	$Ra^{1/3}$	$Ra^{4/9}Pr^{-2/3}$

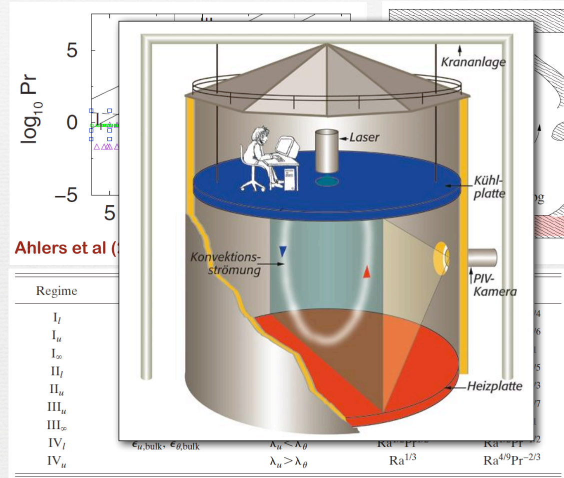
Grossmann & Lohse (2000,2001, 2002, 2004)

For L ~ 7m (Barrel of Ilmenau)

λ ~ 1 mm for Ra = 10^{14}

16

Plumes and *Boundary Layers!*



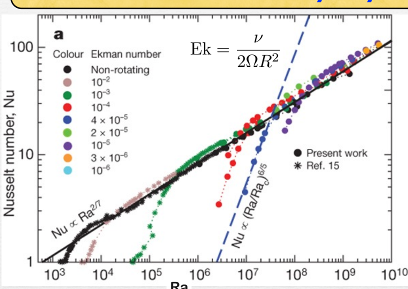
Grossmann & Lohse (2000,2001, 2002, 2004)

For L ~ 7m (Barrel of Ilmenau)

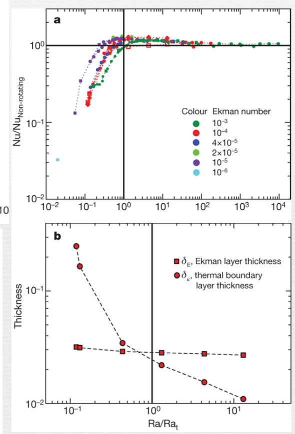
λ ~ 1 mm for Ra = 10^{14}

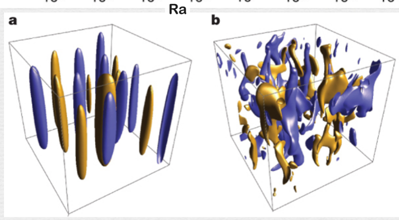
16

Plumes and *Boundary Layers* with *Rotation*



King et al (2009)





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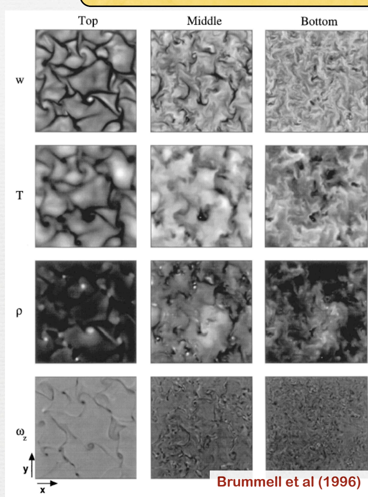
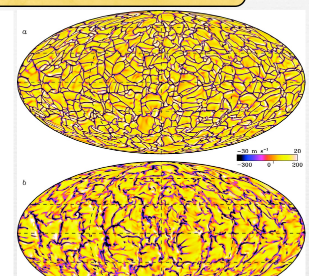
Density Stratification: *Downflow lanes and plumes*



Brummell et al (1996)

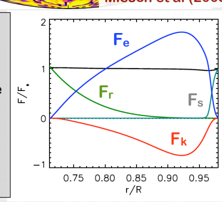


Miesch et al (2008)

Fast, turbulent downflows

slower, more laminar upflows

Inward KE flux



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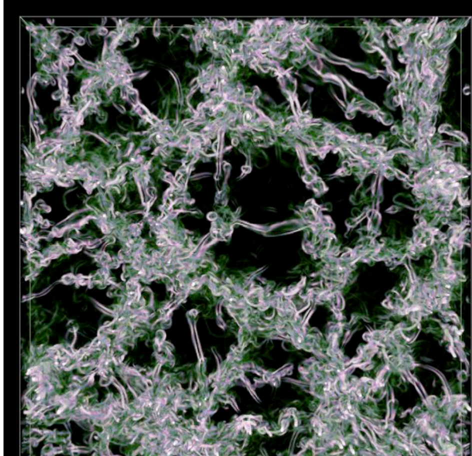
Density Stratification: *Turbulent downflows*



Vortex stretching

Shear

Entrainment

Compression

$$\frac{\partial \omega^2}{\partial t} = \ldots + \omega^2 \nabla\cdot\mathbf{v}$$
$$\approx \ldots - \omega^2 \frac{v_r}{H_\rho}$$

Porter & Woodward (2000)

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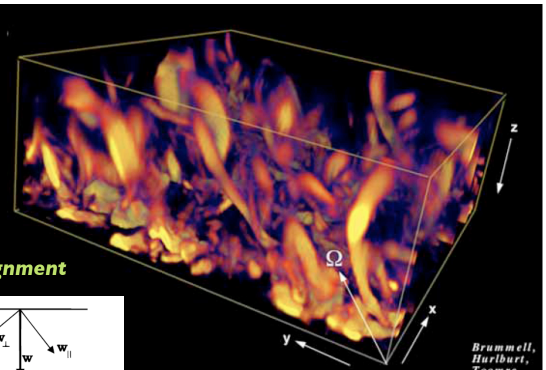
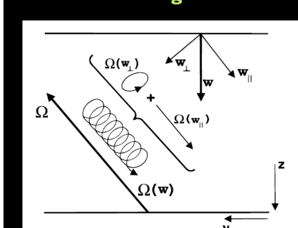
Rotation + Density Stratification: *Helical Downflows*



Brummell, Hurlburt, Toomre

Turbulent Alignment



Rossby Number

$$\mathrm{Ro} = \frac{\omega_{rms}}{2\Omega}$$

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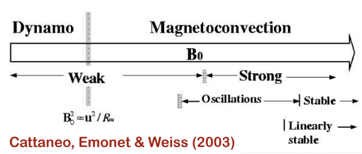
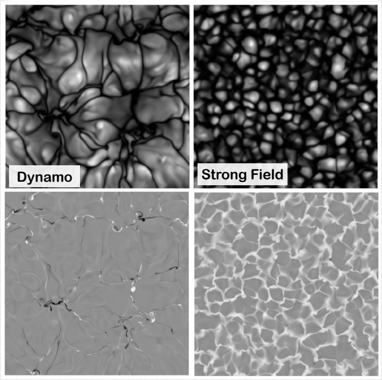
Magnetism: Field Amplification and Advection

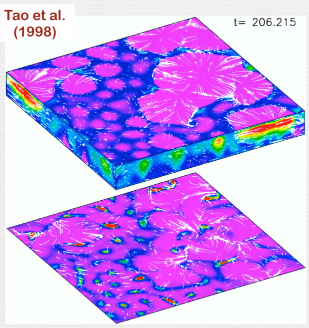
Flux Expulsion
Flux Separation
Turbulent Diamagnetism
Magnetic Pumping
Subcritical instability



Cattaneo, Emonet & Weiss (2003)



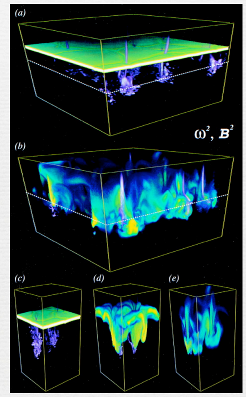
Tao et al. (1998)

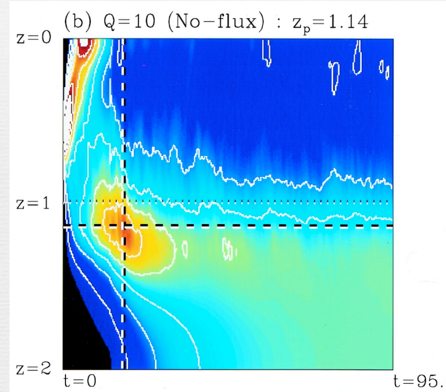


Magnetism: Magnetic Pumping

Why downward transport?
Flow asymmetry (downflows are faster)
Topological connectivity (Moffatt 1978)

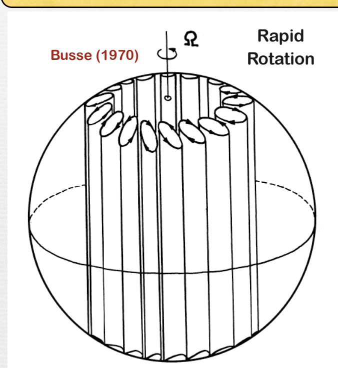
Tobias et al. (2001)



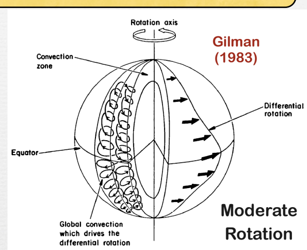


Spherical Geometry: Convective Columns, Tangent Cylinder

Busse (1970)



Gilman (1983)

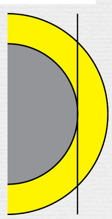


Busse Columns
Banana Cells
Thermal Rossby Waves

In convective shells, columnar convection modes only exist outside the tangent cylinder

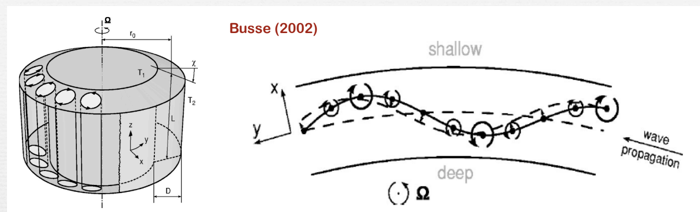
Delineates two distinct convection regimes:

Equatorial modes
Polar Modes



Spherical Geometry: Thermal Rossby Waves

Busse (2002)



Potential Vorticity

$$Q = \frac{\omega_z + 2\Omega}{H\rho}$$

$$\frac{DQ}{Dt} = 0$$

anelastic, adiabatic motions, inviscid, non-magnetic, Ro << 1, $\Omega \cdot \nabla\rho = 0$

(Glatzmaier & Gilman 1981)

Can be driven either by the spherical curvature of the outer boundary or by the density stratification

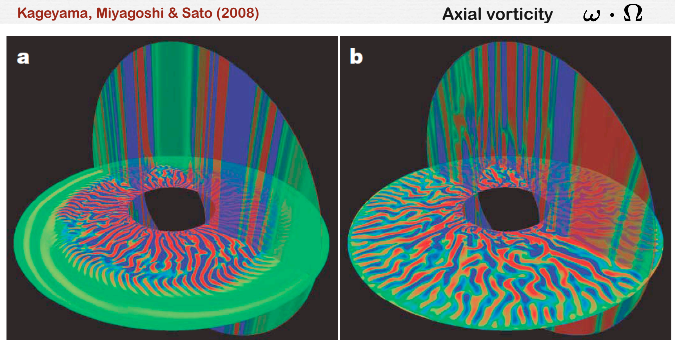
Simplest example: Boussinesq fluid, centrifugal gravity, local, linear perturbations, small boundary curvature (Busse 2002)

$$v_p = \frac{4\Omega}{L}\frac{\tan\chi}{(1+P_r)(k_y^2+k_x^2)}$$

Turbulent convection in Rapidly-Rotating systems

Kageyama, Miyagoshi & Sato (2008)

Axial vorticity $\omega \cdot \Omega$



$\mathrm{Ek} = 2.3 \times 10^{-7}$ $\mathrm{Ek} = 2.6 \times 10^{-6}$

Busse columns give way to vortex sheets but the flow is still approximately 2D

$$\mathrm{Ek} = \frac{\nu}{2\Omega R^2}$$

Summary: Key Concepts

- Plumes
  - Characteristic feature of turbulent convection (lab, simulations, stars...)
  - Density stratification introduces asymmetry: downflows stronger
  - Rotation imparts helicity (sign = $\hat{\Omega} \cdot \hat{g}$): cyclonic downflows
  - Rotation imparts tilt: Turbulent alignment
- Boundary Layers
  - In the lab, this is the way the fluid "feels" the thermal (heating/cooling) and mechanical (rotation) driving (what about stars?)
  - Strong influence on dynamics throughout the domain despite their small extent
- Magnetism
  - Weak fields amplified by convection: dynamo action
  - Intermediate fields pushed aside by convection: flux separation, magnetic pumping
  - Strong fields suppress convection
- Spherical Geometry
  - Convective Columns
  - Tangent Cylinder
  - Thermal Rossby Waves

NCAR

CU, Jan, 2012