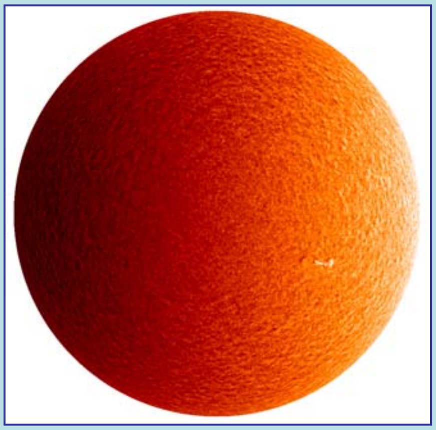
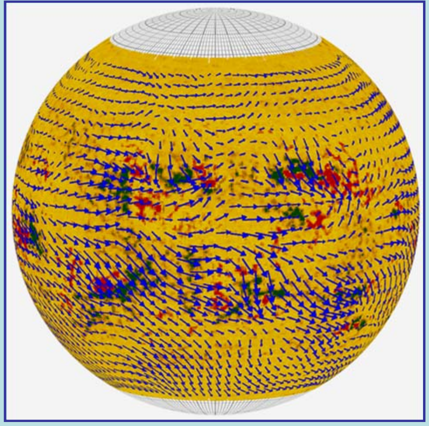
ASTR 7500: Solar & Stellar Magnetism

Hale CGEP Solar & Space Physics





Profs. Brad Hindman & Juri Toomre

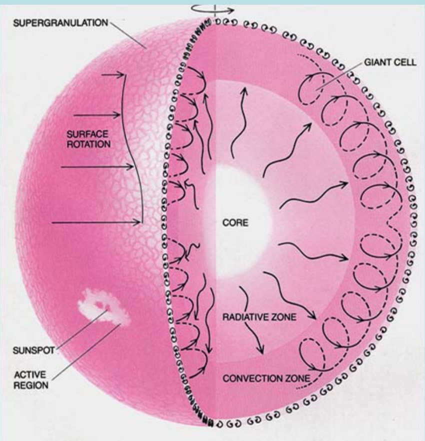
Lecture 21 Tues 9 Apr 2013

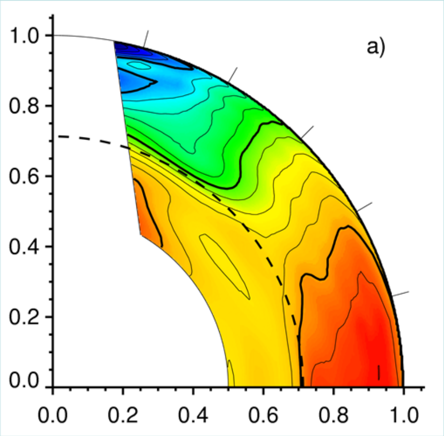
zeus.colorado.edu/astr7500-toomre

Lecture 21
Helioseismology

- Observations of Solar Oscillations
  - Instruments
  - Dopplergrams
  - Power spectra
- Solar Wave Cavities
  - Restoring forces
  - Acoustic waves
  - Gravity Waves
- Wave Excitation and Resonances
  - Granulation makes waves
  - Resonant oscillations
  - Eigenfunctions

Triumphs of Helioseismology





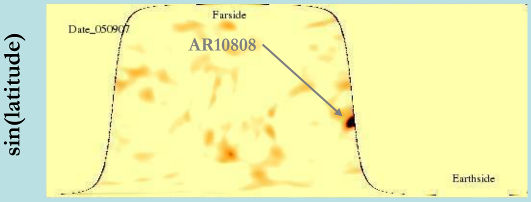
The solar neutrino problem

The sun's internal rotation rate

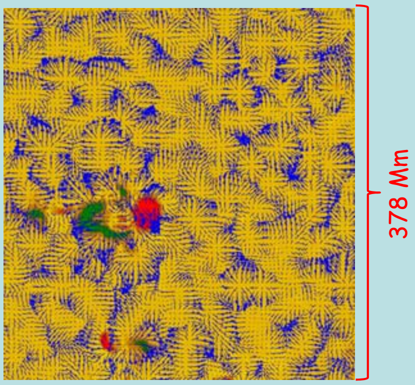
Acoustic Tomography

Active Region NOAA-10808

Aug 29 → Sep 9 2005 (GONG)



Carrington Longitude



Acoustic imaging of the magnetic field on the farside of the sun

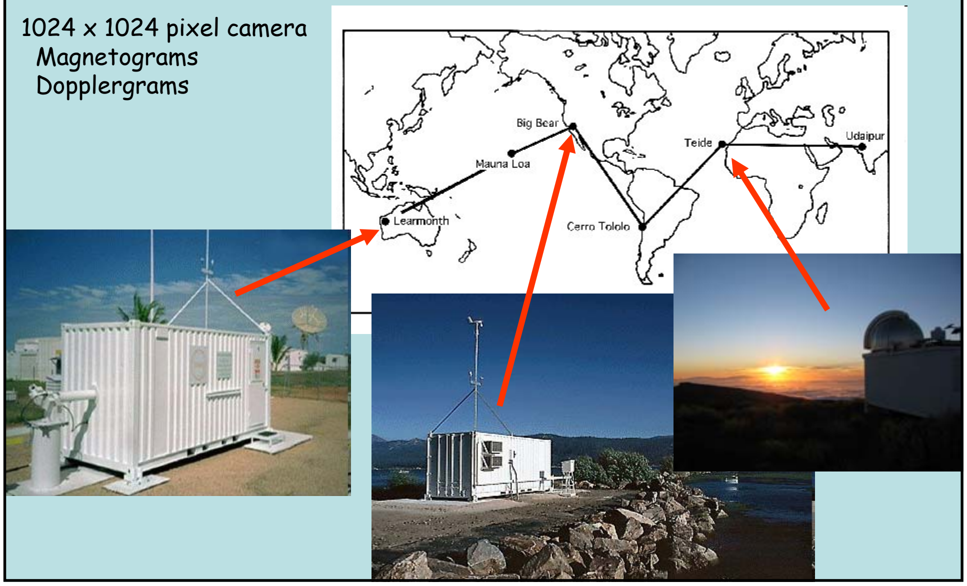
Fine scale measurements of subsurface convection

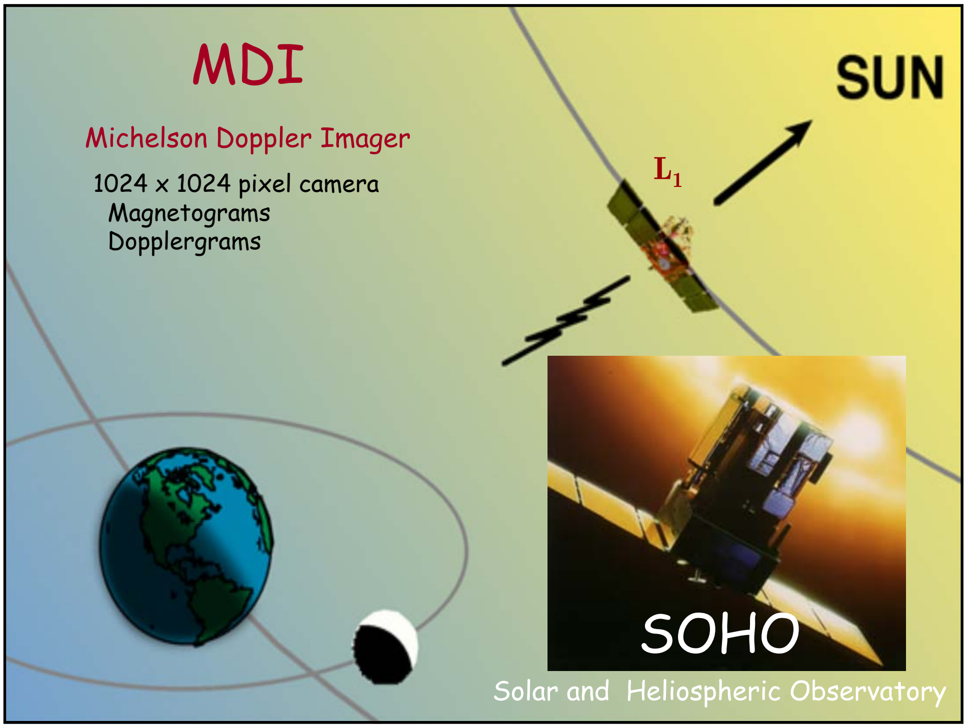
Observations of Solar Oscillations

GONG

Global Oscillation Network Group

1024 x 1024 pixel camera
  Magnetograms
  Dopplergrams





MDI

Michelson Doppler Imager

1024 x 1024 pixel camera
Magnetograms
Dopplergrams

SOHO

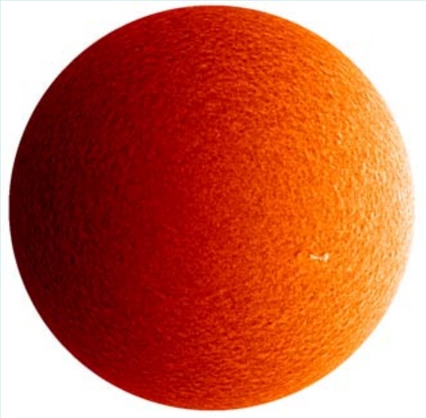
Solar and Heliospheric Observatory

HMI

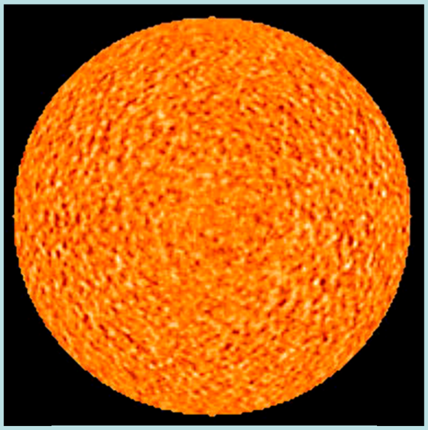
Helioseismic and Magnetic Imager

4096 x 4096 pixel camera
Vector Magnetograms
Dopplergrams

Dopplergrams (MDI)



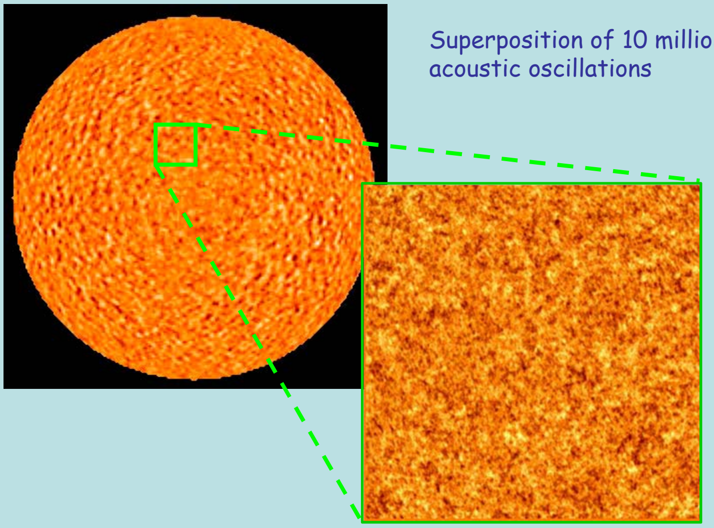
Dopplergram of the Sun



Dopplergram sequence with
Rotation removed

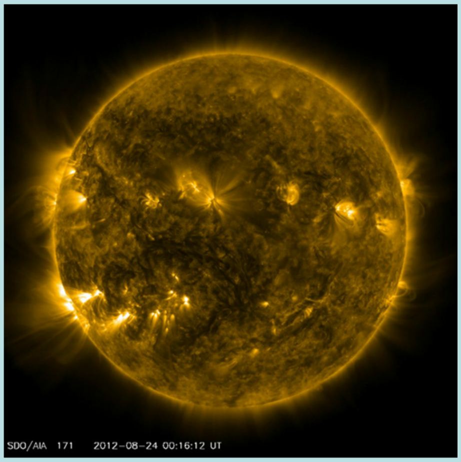
Dopplergram Movies

Superposition of 10 million resonant
acoustic oscillations

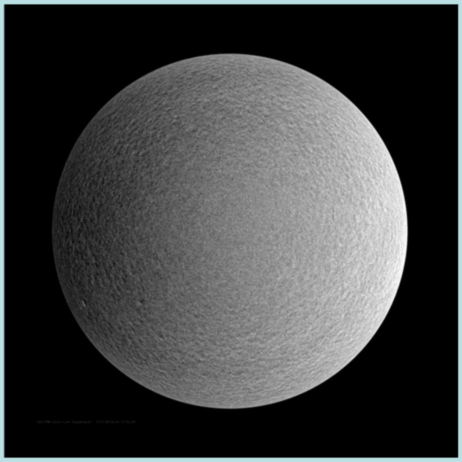


New HMI & AIA Images

From August 24th 2012

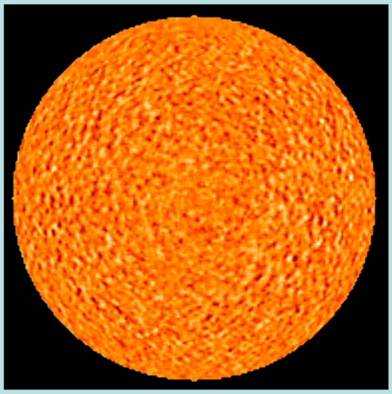


AIA 171 Å
(Low Corona ~ 6×10^5 K)



HMI Dopplergrams
(Photosphere ~ 5000 K)

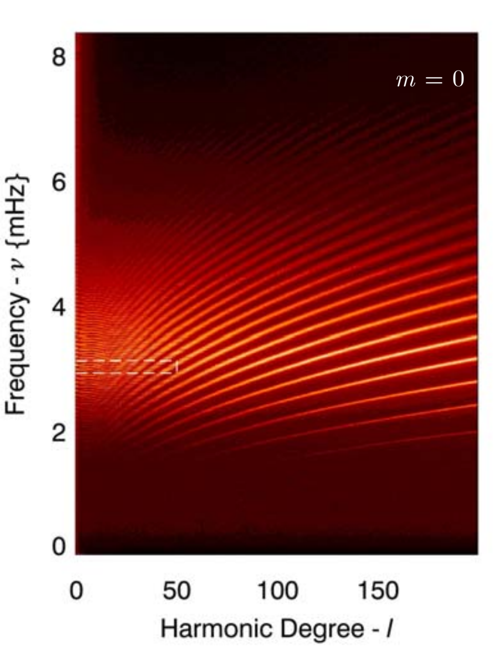
Observations
Spherical Coordinates



Dopplergram sequence with
rotation removed

The raw data appears in the form of a time series of rectangular images (x,y). Each image is converted into spherical coordinates.

$$v_{\mathrm{los}}(x, y, t) \Rightarrow v_{\mathrm{los}}(\varphi, \theta, t)$$



Acoustic Power Spectra

spherical harmonic transforms

$$v_{lm} = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta \;\; Y_l^m(\theta,\varphi)^* \; v_{\text{los}}(\theta,\varphi)$$

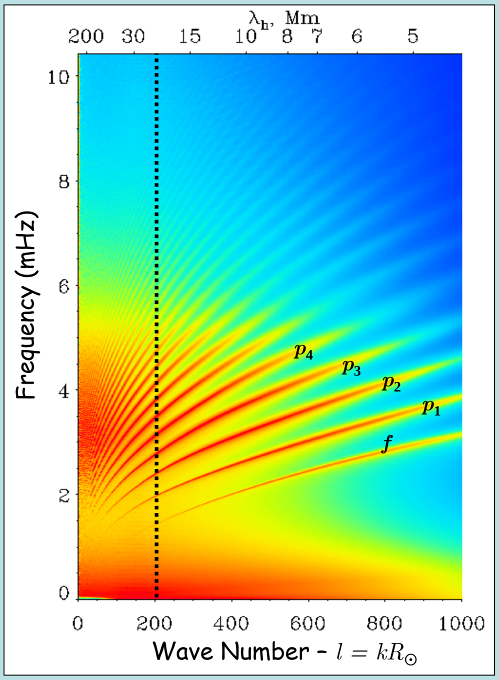
Fourier transforms in time

$$v_{lm}(\nu) = \int dt \;\; e^{-2\pi i \nu t} \; v_{lm}(t)$$

Power spectra

$$P_{lm}(\nu) = \left| v_{lm}(\nu) \right|^2$$

GONG (Global Oscillations Network Group)
3 months of Dopplergrams
(line-of-sight velocity)



Wave Power

$$P_{lm}(\nu) = \left| v_{lm}(\nu) \right|^2$$

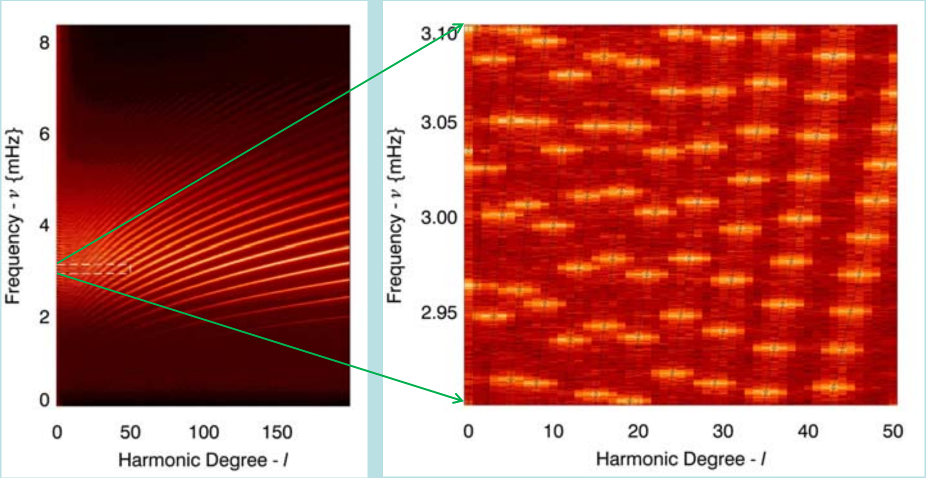
The harmonic degree is proportional to the horizontal wavenumber

Power is concentrated in discrete ridges

Each ridge corresponds to a wave with a different number of radial nodes

Low Harmonic Degree

If we look carefully, we can see that each ridge is actually comprised of individual modes



Solar Wave Cavities

Momentum Equation

Consider the inviscid form of the Navier-Stokes equation, in the presence of gravity in a rotating reference frame.

$$\rho\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\vec{\nabla}P + \rho\left(\vec{g}_{\mathrm{N}} + \Omega^2\vec{R}\right) - 2\rho\,\vec{\Omega}\times\vec{v}$$

Pressure perturbations drive sound waves

Buoyancy perturbations drive internal gravity waves

The Coriolis force drives Rossby waves

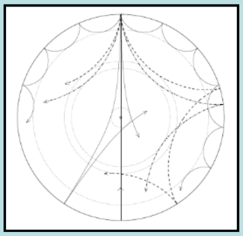
We won't spend much attention on such issues

$$\vec{g} \equiv \vec{g}_{\mathrm{N}} + \Omega^2\vec{R}$$

Newtonian gravity and the centrifugal force can be combined into an effective gravity

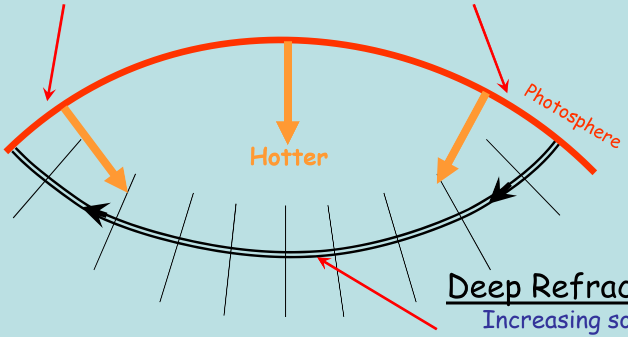
How are sound waves trapped?

Acoustic waves are trapped within a spherical shell



Surface Reflection

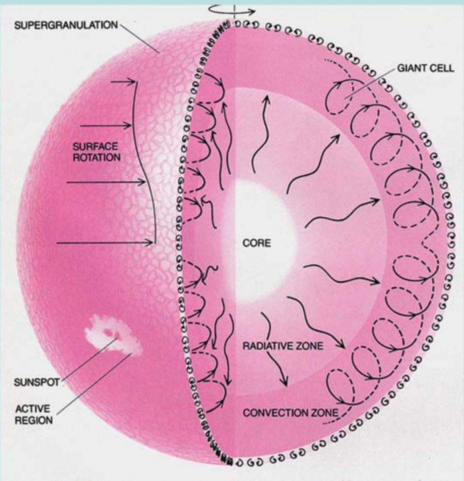
Waves reflect off the stellar photosphere because of the rapid decrease in mass density.



Deep Refraction

Increasing sound speed with depth refracts waves back towards the surface

How are gravity waves trapped?



In the convection zone

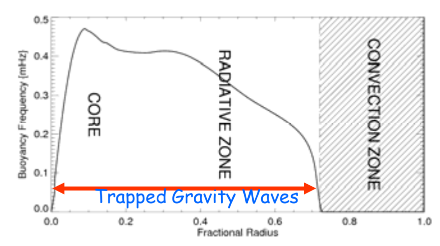
$$N^2 \approx 0$$

In the core and radiative zones

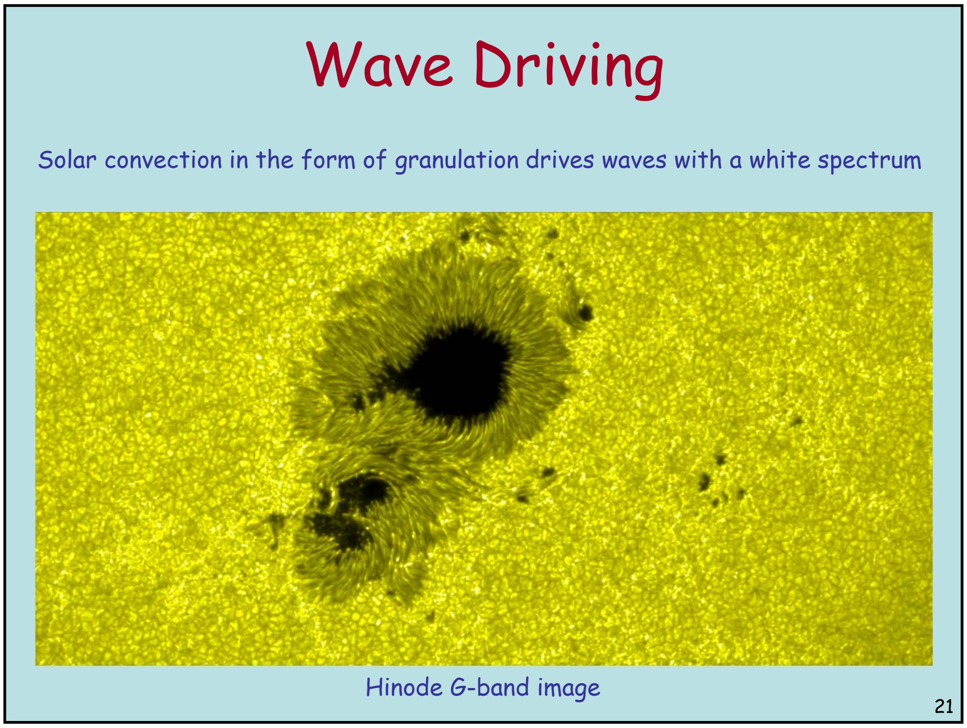
$$N^2 > 0$$

Gravity waves propagate if

$$\omega^2 < N^2$$



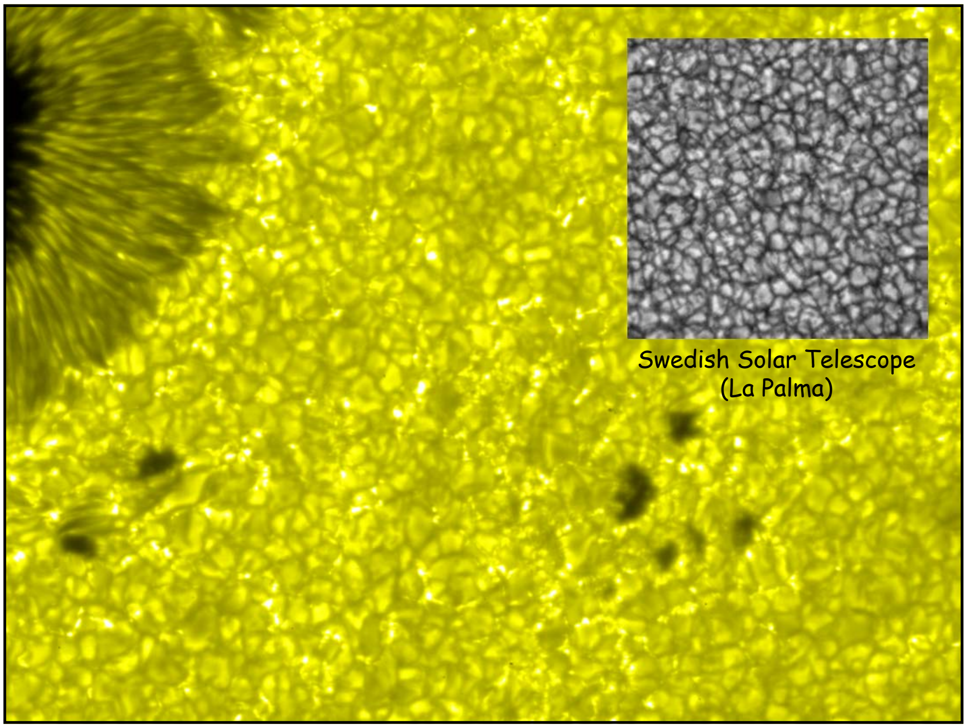
Excitation and Resonances



Wave Driving

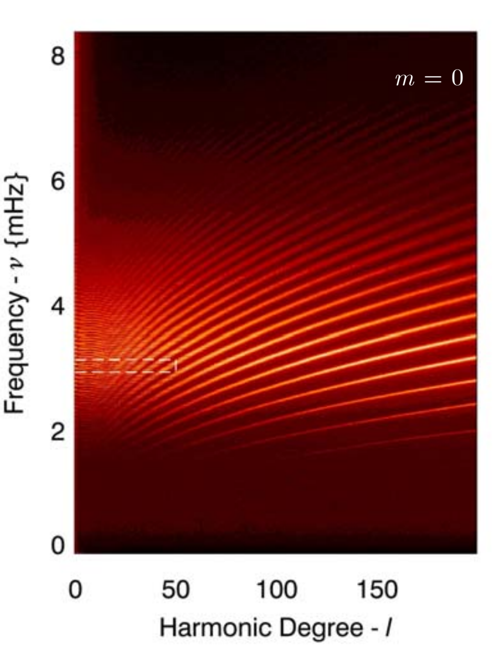
Solar convection in the form of granulation drives waves with a white spectrum

Hinode G-band image

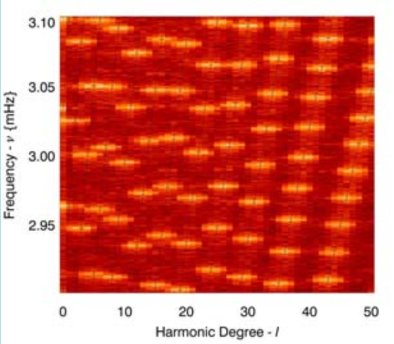


Swedish Solar Telescope
(La Palma)

Acoustic Power Spectra



If the forcing is over a broad spectrum, why is the observed energy confined to special frequencies?



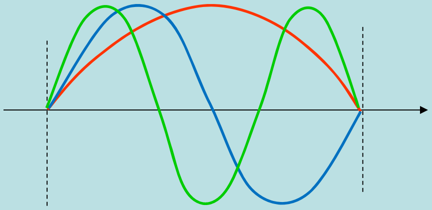
Resonances

Since we have trapped waves, we can have resonant oscillations.

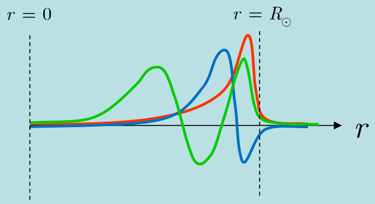
Constructive self interference amplifies the energy for the resonant frequencies.

The resonant oscillations satisfy an eigenproblem.

- Wave equation
- Boundary conditions



First few modes of a guitar string



First few modes of the sun

Form of the Eigenfunctions

For spherical symmetric stars the eigenfunction is separable

$$w(r,\phi,\theta,t) = W(r)\, Y(\phi,\theta)\, e^{-i\omega t}$$

Radial Eigenfunction

Angular Eigenfunction

The radial and angular eigenfunctions satisfy separate wave equations

$$\nabla_{\mathrm{h}}^2 Y(\phi,\theta) + \frac{l(l+1)}{r^2} Y(\phi,\theta) = 0$$

Horizontal Wavenumber $k_{\mathrm{h}}^2 = \frac{l(l+1)}{r^2}$

$$\mathcal{L}\{W\}(\omega,r) + \frac{l(l+1)}{r^2} W(r) = 0$$

Fourth Order

ω is the eigenvalue

Separation Constant $l(l+1)$

Spherical Harmonics

The ODE for the angular function Y is well-known and its solutions are functions called spherical harmonics

Associated Legendre Polynomials

Fourier Modes

$$Y = Y_l^m(\phi,\theta)$$

$$= (-1)^m C_{lm} P_l^m(\cos\theta)\, e^{im\phi}$$

<u>Angular Boundary Conditions</u>

There are two separation constants (l and m) and both have integer values.

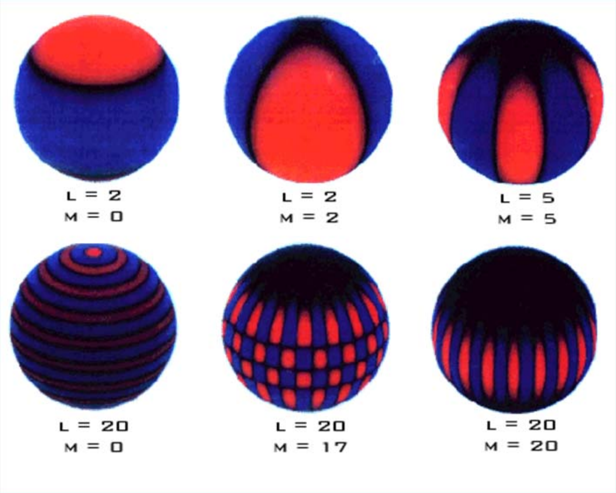
l must be an integer for the solution to be finite at the poles

m must be an integer for the solution to be continuous at $\phi = 0$ and 2π

Furthermore $|m| \leq l$, $l = 1$ has a triplet $m = \{-1, 0, 1\}$

$l = 2$ has a quintuplet $m = \{-2, -1, 0, 1, 2\}$

Quantum Numbers



l determines the horizontal scale

$$k_{\rm h}^2 = \frac{l(l+1)}{r^2}$$

$$\nabla_{\rm h}^2 Y = -k_{\rm h}^2 Y$$

m determines the number of nodes in longitude

$l - m$ determines the number of nodes in latitude

$m\ /\ l$ determines how close to the pole

Acknowledgements: BISON group

Radial Order

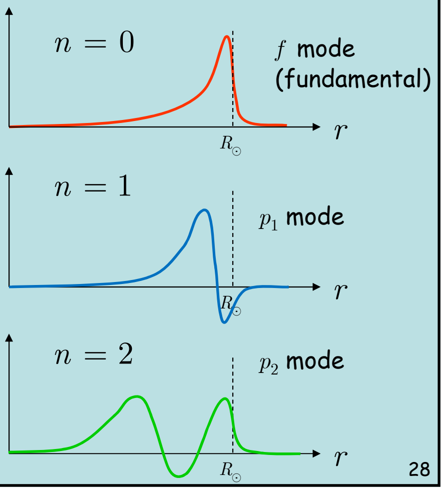
$$\mathcal{L}\{W_n\}(\omega, r) + k_{\rm h}^2 W_n(r) = 0$$

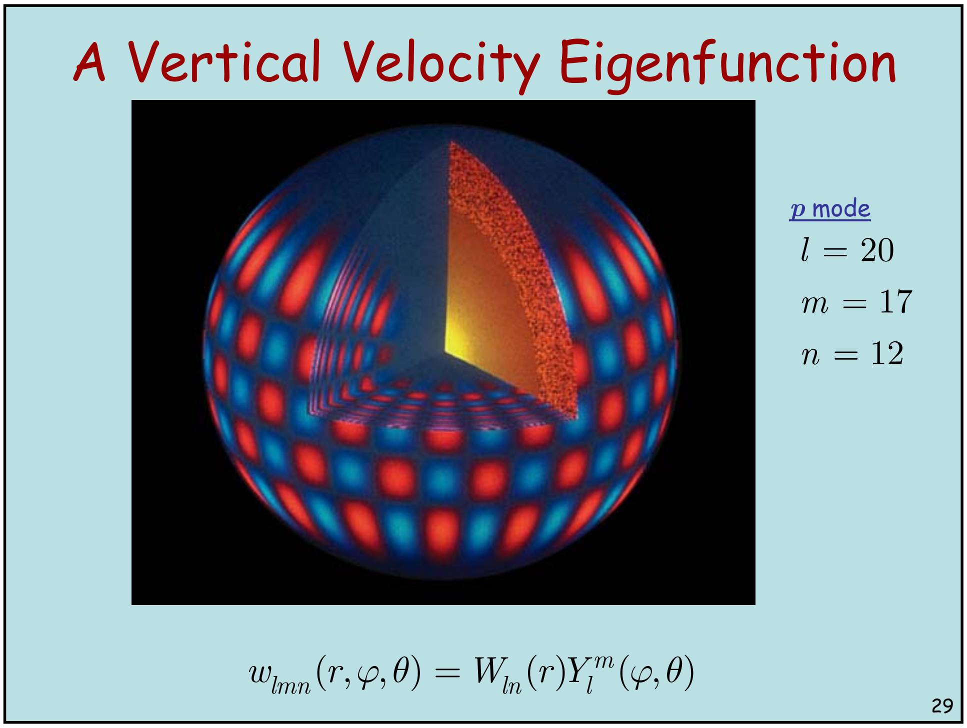
Radial Order n

The radial differential equation permits a sequence of solutions with different numbers of nodes.

The eigenfunctions are labeled by the number of nodes n.

Each solution has a different corresponding eigenfrequency ω_{lmn}.





A Vertical Velocity Eigenfunction

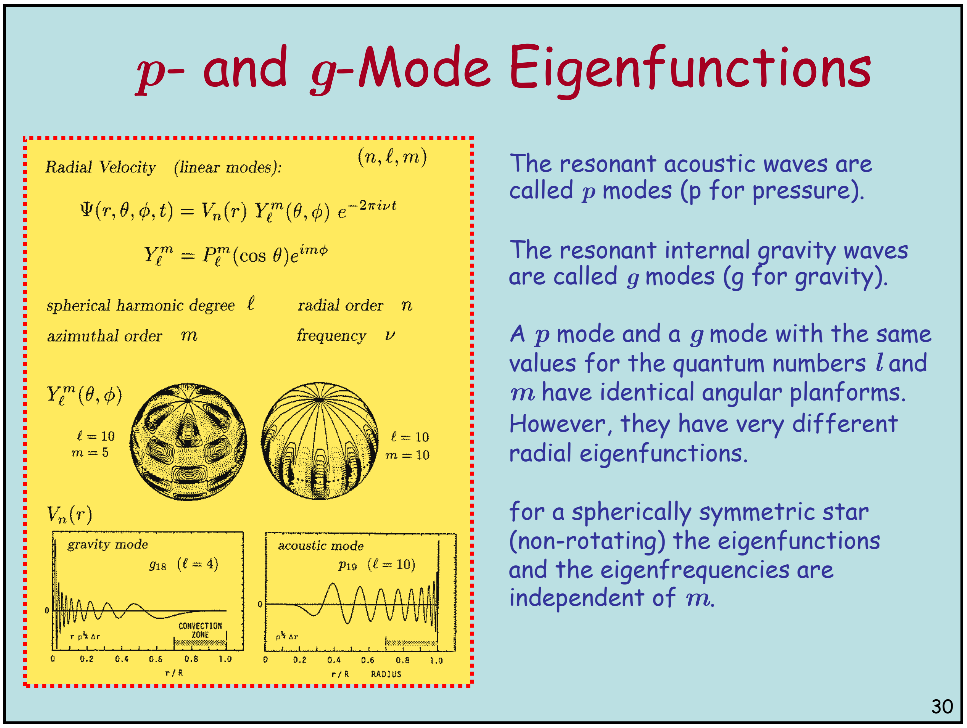
p mode

$l = 20$

$m = 17$

$n = 12$

$$w_{lmn}(r,\varphi,\theta) = W_{ln}(r)Y_l^m(\varphi,\theta)$$



p- and g-Mode Eigenfunctions

Radial Velocity (linear modes): (n,ℓ,m)

$$\Psi(r,\theta,\phi,t) = V_n(r)\ Y_\ell^m(\theta,\phi)\ e^{-2\pi i\nu t}$$

$$Y_\ell^m = P_\ell^m(\cos\,\theta)e^{im\phi}$$

spherical harmonic degree ℓ

radial order n

azimuthal order m

frequency ν

$Y_\ell^m(\theta,\phi)$

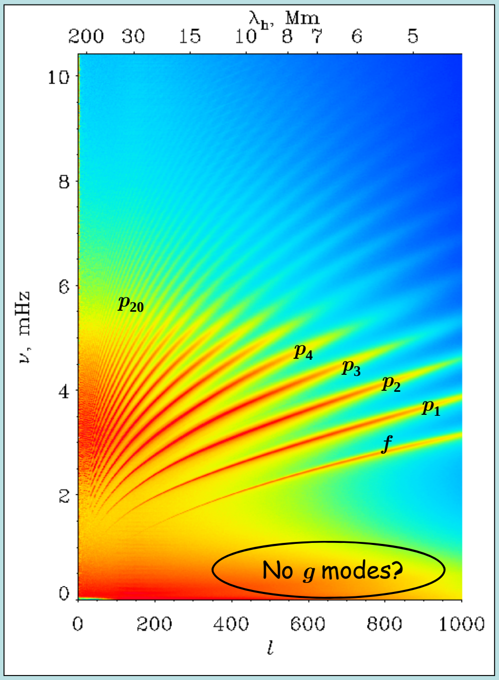
$V_n(r)$

The resonant acoustic waves are called p modes (p for pressure).

The resonant internal gravity waves are called g modes (g for gravity).

A p mode and a g mode with the same values for the quantum numbers l and m have identical angular planforms. However, they have very different radial eigenfunctions.

for a spherically symmetric star (non-rotating) the eigenfunctions and the eigenfrequencies are independent of m.



Power Spectrum

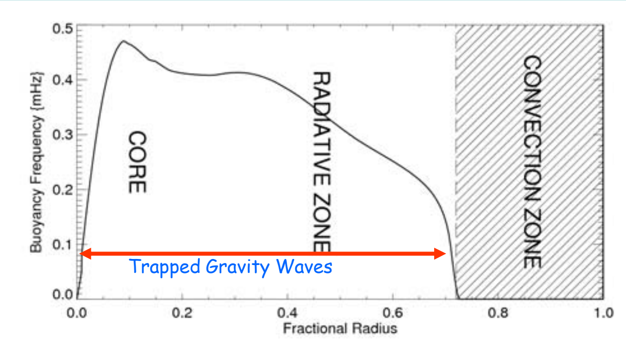
f - **Fundamental** mode
(surface gravity wave)

p_n - acoustic mode
(**P**ressure wave)

g_n - gravity mode
(internal **G**ravity wave)

Why are only the acoustic modes visible?

Tunneling



g modes must tunnel through the convection zone before they can be observed at the surface.

Thus, they have very low amplitude and have remained unobserved (in the sun).