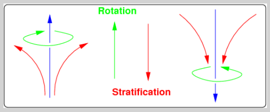
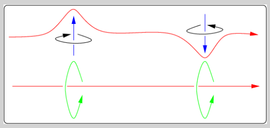
ASTR 7500: Solar & Stellar Magnetism

Hale CGEG Solar & Space Physics





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Lecture 11 Tues 26 Feb 2013

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Summary: Mean field theory

Average of induction equation:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\mathbf{v}' \times \mathbf{B}'} + \overline{\mathbf{v}} \times \overline{\mathbf{B}} - \eta \boldsymbol{\nabla} \times \overline{\mathbf{B}} \right)$$

New solution properties arie from the term:

$$\overline{\mathcal{E}} = \overline{\mathbf{v}' \times \mathbf{B}'}$$

Assumption of scale separation in time and space:

$$\overline{\mathcal{E}}_i = a_{ij} \overline{B}_j + b_{ijk} \frac{\partial \overline{B}_j}{\partial x_k}$$

Summary: Mean field theory

Some reordering of terms:

$$\overline{\mathcal{E}} = \boldsymbol{\alpha} \overline{\mathbf{B}} + \boldsymbol{\gamma} \times \overline{\mathbf{B}} - \boldsymbol{\beta} \boldsymbol{\nabla} \times \overline{\mathbf{B}} - \boldsymbol{\delta} \times \boldsymbol{\nabla} \times \overline{\mathbf{B}} + \dots$$

- α, β: symmetric tensors
- γ, δ: vectors

Symmetry constraints imply:

- $\boldsymbol{\alpha}$, $\boldsymbol{\delta}$: pseudo tensor (related to helicity and rotation)
- $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$: true tensors

Summary: Mean field theory

Assumption isotropy (non mirror-symmetric, weakly inhomogeneous):

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \boldsymbol{\nabla} \times \left[\alpha \overline{\mathbf{B}} + (\overline{\mathbf{v}} + \boldsymbol{\gamma}) \times \overline{\mathbf{B}} - (\eta + \eta_t) \boldsymbol{\nabla} \times \overline{\mathbf{B}} \right]$$

with the scalar quantities

$$\alpha = -\frac{1}{3} \tau_c \overline{\mathbf{v}' \cdot (\boldsymbol{\nabla} \times \mathbf{v}')}, \qquad \eta_t = \frac{1}{3} \tau_c \overline{\mathbf{v}'^2}$$

and vector

$$\boldsymbol{\gamma} = -\frac{1}{6} \tau_c \boldsymbol{\nabla} \overline{\mathbf{v}'^2} = -\frac{1}{2} \boldsymbol{\nabla} \eta_t$$

Turbulent diffusivity - destruction of magnetic field

Turbulent diffusivity dominant dissipation process for large scale field in case of large R_m:

$$\eta_t = \frac{1}{3} \tau_c \overline{\mathbf{v}'^2} \sim L \, v_{\mathrm{rms}} \sim R_m \eta \gg \eta$$

- Formally η_t comes from advection term (transport term, non-dissipative)
- Turbulent cascade transporting magnetic energy from the large scale L to the micro scale l_m (advection + reconnection)

$$\eta \mathbf{j}_m^2 \sim \eta_t \overline{\mathbf{j}}^2 \longrightarrow \frac{B_m}{l_m} \sim \sqrt{R_m} \frac{\overline{B}}{L}$$

Important: The large scale determines the energy dissipation rate, l_m adjusts to allow for the dissipation on the microscale.
Present for isotropic homogeneous turbulence

Turbulent diamagnetism, turbulent pumping

Expulsion of flux from regions with larger turbulence intensity 'diamagnetism'

$$\boldsymbol{\gamma} = -\frac{1}{2} \boldsymbol{\nabla} \eta_t$$

Turbulent pumping (stratified convection):

$$\boldsymbol{\gamma} = -\frac{1}{6} \tau_c \boldsymbol{\nabla} \overline{\mathbf{v}'^2}$$

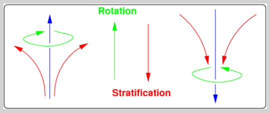
- Upflows expand, downflows converge
- Stronger velocity and smaller filling factor of downflows
- Mean induction effect of up- and downflow regions does not cancel
- Downward transport found in numerical simulations

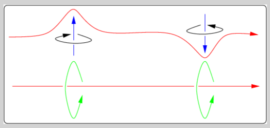
Requires inhomogeneity (stratification)

Kinematic α-effect

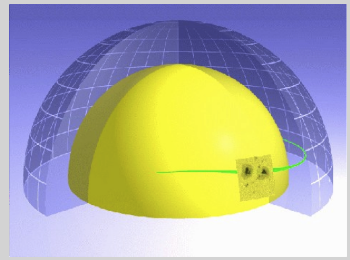
$$\alpha = -\frac{1}{3}\tau_c \overline{\mathbf{v}' \cdot (\nabla \times \mathbf{v}')} \qquad H_k = \overline{\mathbf{v}' \cdot (\nabla \times \mathbf{v}')} \text{ kinetic helicity}$$

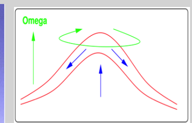
Requires rotation + additional preferred direction (stratification)

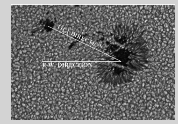




Babcock-Leighton α-effect







- Similar to kinetic α-effect, but driven by magnetic buoyancy
- Leading polarities have larger propability to reconnect across equator with counterpart on other hemisphere
- Polarity of hemisphere = polarity of following sunspots

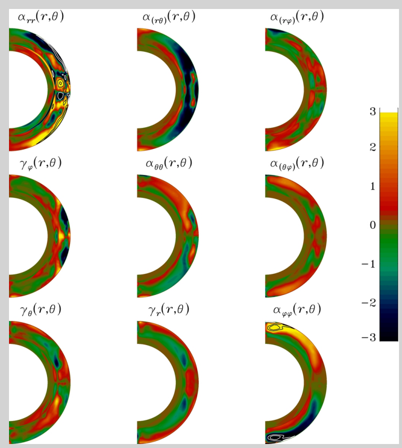
Fast or slow dynamo?

Turbulent induction effects require reconnection to operate; however, the expressions

$$\begin{aligned}
\alpha_{ij} &= \frac{1}{2}\tau_c \left(\varepsilon_{ikl} \overline{v_k{}' \frac{\partial v_l{}'}{\partial x_j}} + \varepsilon_{jkl} \overline{v_k{}' \frac{\partial v_l{}'}{\partial x_i}} \right) \\
\gamma_i &= -\frac{1}{2}\tau_c \frac{\partial}{\partial x_k} \overline{v_i' v_k'} \\
\beta_{ij} &= \frac{1}{2}\tau_c \left(\overline{\mathbf{v}'^2} \delta_{ij} - \overline{v_i{}' v_j{}'} \right)
\end{aligned}$$

are independent of η (in this approximation), indicating fast dynamo action (no formal proof since we made strong assumptions!)

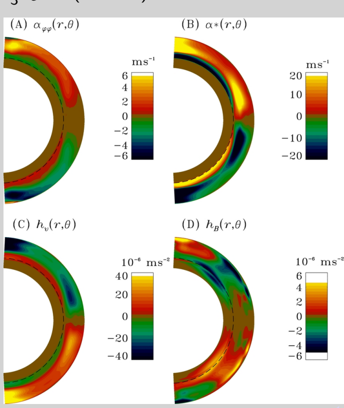
How well does this work in practice?



From Racine et al. 2011

How well does this work in practice?

$\alpha = -\frac{1}{3}\tau_c \overline{\mathbf{v}' \cdot (\nabla \times \mathbf{v}')}$



From Racine et al. 2011

Generalized Ohm's law

What is needed to circumvent Cowling's theorem?

- Crucial for Cowling's theorem: Impossibility to drive a current parallel to magnetic field
- Cowling's theorem does not apply to mean field if a mean current can flow parallel to the mean field (since total field non-axisymmetric this is not a contradiction!)

$$\bar{\mathbf{j}} = \tilde{\boldsymbol{\sigma}} \left(\bar{E} + \bar{\mathbf{v}} \times \bar{\mathbf{B}} + \boldsymbol{\gamma} \times \bar{\mathbf{B}} + \boldsymbol{\alpha} \bar{\mathbf{B}} \right)$$

$\tilde{\boldsymbol{\sigma}}$ contains contributions from η, β and δ.
Ways to circumvent Cowling:

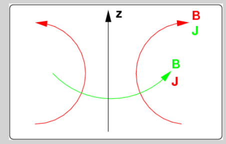
- α-effect
- anisotropic conductivity (off diagonal elements + δ-effect)

Meanfield energy equation

$$\frac{d}{dt}\int \frac{\overline{\mathbf{B}}^2}{2\mu_0}\,dV = -\mu_0 \int \eta \overline{\mathbf{j}}^2\,dV - \int \overline{\mathbf{v}}\cdot(\overline{\mathbf{j}}\times\overline{\mathbf{B}})\,dV + \int \overline{\mathbf{j}}\cdot\overline{\mathcal{E}}\,dV$$

- Energy conversion by α-effect $\sim \alpha\overline{\mathbf{j}}\cdot\overline{\mathbf{B}}$
- α-effect only pumps energy into meanfield if meanfield is helical (current helicity must have same sign as α)!
- Dynamo action does not necessarily require that $\overline{\mathbf{j}}\cdot\overline{\mathcal{E}}$ is an energy source. It can be sufficient if $\overline{\mathcal{E}}$ changes field topology to circumvent Cowling, if other energy sources like differential rotation are present (i.e. $\Omega\times\overline{\mathbf{j}}$ effect).

α^2-dynamo



Induction of field parallel to current (producing helical field!)

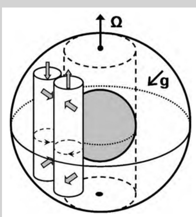
$$\frac{\partial\overline{\mathbf{B}}}{\partial t} = \nabla\times(\alpha\overline{\mathbf{B}}) = \alpha\mu_0\overline{\mathbf{j}}$$

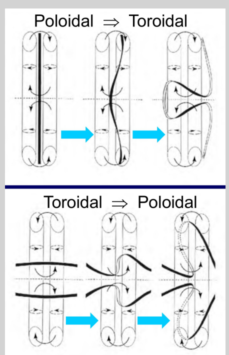
Dynamo cycle:

$$\mathbf{B}_t \xrightarrow{\alpha} \mathbf{B}_p \xrightarrow{\alpha} \mathbf{B}_t$$

- Poloidal and toroidal field of similar strength
- In general stationary solutions

α^2 model for Geodynamo

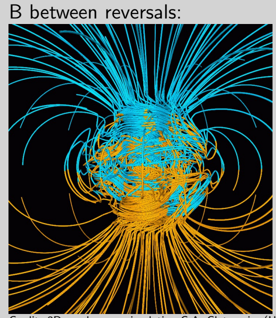




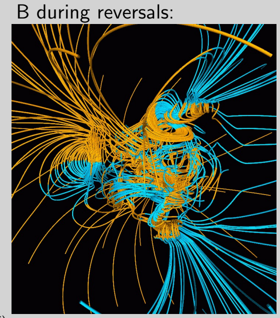
- Strong influence of rotation
  $\tau_{rot} = 1$ day, $\tau_c \sim 1000$ years
  (Sun: $\tau_{rot} = 27$ days, $\tau_c \sim$ weeks)
- Flow organization: Taylor columns
- Secondary flow along columns (boundary effect) $\longrightarrow$ helicity

α^2 model for Geodynamo

B between reversals:

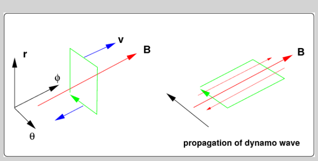


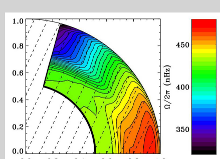
B during reversals:



Credit: 3D geodynamo simulation G.A. Glatzmaier (UCSC)

$\alpha\Omega$-, $\alpha^2\Omega$-dynamo





Dynamo cycle:

$$\mathbf{B}_t \xrightarrow{\alpha} \mathbf{B}_p \xrightarrow{\Omega,\,\alpha} \mathbf{B}_t$$

- Toroidal field much stronger that poloidal field
- In general traveling (along lines of constant Ω) and periodic solutions

Movie α-effect Movie Ω-effect

$\alpha\Omega$-dynamo

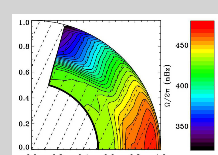
$$\frac{\partial B}{\partial t} = r\sin\mathbf{B}_p\cdot\nabla\Omega + \eta_t\left(\Delta - \frac{1}{(r\sin\theta)^2}\right)B$$

$$\frac{\partial A}{\partial t} = \alpha B + \eta_t\left(\Delta - \frac{1}{(r\sin\theta)^2}\right)A$$

- Cyclic behavior:

$$P \propto (\alpha|\nabla\Omega|)^{-1/2}$$

- Propagation of magnetic field along contourlines of Ω "dynamo-wave"
- Direction of propagation "Parker-Yoshimura-Rule":



$$\mathbf{s} = \alpha\nabla\Omega\times\mathbf{e}_\phi$$

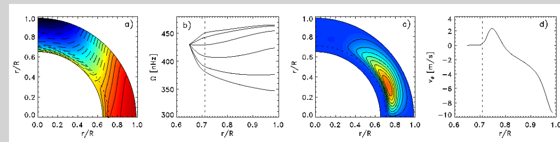
Movie: $\alpha\Omega$-dynamo

$\alpha\Omega$-dynamo with meridional flow

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_r B) + \frac{\partial}{\partial \theta}(v_\theta B)\right) = r \sin \mathbf{B}_p \cdot \nabla\Omega$$
$$+ \eta_t \left(\Delta - \frac{1}{(r\sin\theta)^2}\right) B$$
$$\frac{\partial A}{\partial t} + \frac{1}{r\sin\theta}\mathbf{v}_p \cdot \nabla(r\sin\theta A) = \alpha B + \eta_t\left(\Delta - \frac{1}{(r\sin\theta)^2}\right) A$$

- If η_t is sufficiently small, such that:
  $$\tau_d = D_{CZ}^2/\eta_t > D_{CZ}/v_m \longrightarrow \eta_t < v_m D_{CZ}$$
  the meridional flow v_m can control the cycle period and propagation of the magnetic activity
- Additional advection like effects can arise from the γ-effect, they can be accounted for by formally substituting:
  $$\mathbf{v}_m \longrightarrow \mathbf{v}_m + \gamma$$

$\alpha\Omega$-dynamo with meridional flow



Meridional flow:
- Poleward at top of convection zone
- Equatorward at bottom of convection zone

Effect of advection:
- Equatorward propagation of activity
- Correct phase relation between poloidal and toroidal field
- Circulation time scale of flow sets dynamo period
- Requirement: Sufficiently low turbulent diffusivity

Movie: Flux-transport-dynamo (M. Dikpati, HAO)

$\Omega \times J$ dynamo

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times [\delta \times (\nabla \times \overline{\mathbf{B}})] \sim \nabla \times (\Omega \times \overline{\mathbf{j}}) \sim \frac{\partial \overline{\mathbf{j}}}{\partial z}$$

- similar to α-effect, but additional z-derivative of current
- couples poloidal and toroidal field
- δ^2 dynamo is not possible:
  $$\overline{\mathbf{j}} \cdot \overline{\mathcal{E}} = \overline{\mathbf{j}} \cdot (\delta \times \overline{\mathbf{j}}) = 0$$
- δ-effect is controversial (not all approximations give a non-zero effect)
- in most situations α dominates

Dynamos and magnetic helicity

Magnetic helicity (integral measure of field topology):

$$H_m = \int \mathbf{A}\cdot\mathbf{B}\, dV$$

has following conservation law (no helicity fluxes across boundaries):

$$\frac{d}{dt}\int \mathbf{A}\cdot\mathbf{B}\, dV = -2\mu_0\eta \int \mathbf{j}\cdot\mathbf{B}\, dV$$

Decomposition into small and large scale part:

$$\frac{d}{dt}\int \overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\, dV = +2\int \overline{\mathcal{E}}\cdot\overline{\mathbf{B}}\, dV - 2\mu_0\eta\int \overline{\mathbf{j}}\cdot\overline{\mathbf{B}}\, dV$$
$$\frac{d}{dt}\int \overline{\mathbf{A}'\cdot\mathbf{B}'}\, dV = -2\int \overline{\mathcal{E}}\cdot\overline{\mathbf{B}}\, dV - 2\mu_0\eta\int \overline{\mathbf{j}'\cdot\mathbf{B}'}\, dV$$

Dynamos and magnetic helicity

Dynamos have helical fields:
- α-effect induces magnetic helicity of same sign on large scale
- α-effect induces magnetic helicity of opposite sign on small scale

Asymptotic staturation (time scale $\sim R_m\tau_c$):

$$\overline{\mathbf{j}'\cdot\mathbf{B}'} = -\overline{\mathbf{j}}\cdot\overline{\mathbf{B}} \longrightarrow \frac{|\overline{B}|}{|B'|} \sim \sqrt{\frac{L}{l_c}}$$
$$\overline{\mathbf{j}'\cdot\mathbf{B}'} = -\frac{\alpha\overline{\mathbf{B}}^2}{\mu_0\eta} + \frac{\eta_t}{\eta}\overline{\mathbf{j}}\cdot\overline{\mathbf{B}}$$

Time scales:
- Galaxy: $\sim 10^{25}$ years ($R_m \sim 10^{18}$, $\tau_c \sim 10^7$ years)
- Sun: $\sim 10^8$ years
- Earth: $\sim 10^6$ years

Non-kinematic effects

Proper way to treat them: 3D simulations
- Still very challenging
- Has been successful for geodynamo, but not for solar dynamo

Semi-analytical treatment of Lorentz-force feedback in mean field models:
- Macroscopic feedback: Change of the mean flow (differential rotation, meridional flow) through the mean Lorentz-force
  $$\overline{\mathbf{f}} = \overline{\mathbf{j}} \times \overline{\mathbf{B}} + \overline{\mathbf{j}' \times \mathbf{B}'}$$
- Mean field model including mean field representation of full MHD equations:
  Movie: Non-kinematic flux-transport dynamo
- Microscopic feedback: Change of turbulent induction effects (e.g. α-quenching)

Microscopic feedback

Feedback of Lorentz force on small scale motions:

- Intensity of turbulent motions significantly reduced if $\frac{1}{2\mu_0}B^2 > \frac{1}{2}\varrho v_{rms}^2$. Typical expression used

$$\alpha = \frac{\alpha_k}{1 + \frac{\overline{\mathbf{B}}^2}{B_{eq}^2}}$$

  with the equipartition field strength $B_{eq} = \sqrt{\mu_0 \varrho} v_{rms}$
- Similar quenching also expected for turbulent diffusivity
- Additional quenching of α due to topological constraints possible (helicity conservation)
  Controversial !

Microscopic feedback

Symmetry of momentum and induction equation $\mathbf{v}' \leftrightarrow \mathbf{B}'/\sqrt{\mu_0\varrho}$:

$$\begin{aligned} \frac{d\mathbf{v}'}{dt} &= \frac{1}{\mu_0\varrho}(\overline{\mathbf{B}}\cdot\nabla)\mathbf{B}' + \ldots \\ \frac{d\mathbf{B}'}{dt} &= (\overline{\mathbf{B}}\cdot\nabla)\mathbf{v}' + \ldots \\ \overline{\boldsymbol{\mathcal{E}}} &= \overline{\mathbf{v}' \times \mathbf{B}'} \end{aligned}$$

Strongly motivates magnetic term for α-effect (Pouquet et al. 1976):

$$\alpha = \frac{1}{3}\tau_c\left(\frac{1}{\varrho}\overline{\mathbf{j}'\cdot\mathbf{B}'} - \overline{\boldsymbol{\omega}'\cdot\mathbf{v}'}\right)$$

- Kinetic α: $\overline{\mathbf{B}} + \mathbf{v}' \longrightarrow \mathbf{B}' \longrightarrow \overline{\boldsymbol{\mathcal{E}}}$
- Magnetic α: $\overline{\mathbf{B}} + \mathbf{B}' \longrightarrow \mathbf{v}' \longrightarrow \overline{\boldsymbol{\mathcal{E}}}$

Microscopic feedback

From helicity conservation one expects

$$\overline{\mathbf{j}'\cdot\mathbf{B}'} \sim -\alpha\overline{\mathbf{B}}^2$$

leading to algebraic quenching

$$\alpha = \frac{\alpha_k}{1 + g\,\frac{\overline{\mathbf{B}}^2}{B_{eq}^2}}$$

With the asymptotic expression (steady state)

$$\overline{\mathbf{j}'\cdot\mathbf{B}'} = -\frac{\alpha\overline{\mathbf{B}}^2}{\mu_0\eta} + \frac{\eta_t}{\eta}\overline{\mathbf{j}}\cdot\overline{\mathbf{B}}$$

we get

$$\alpha = \frac{\alpha_{\rm k} + \frac{\eta_t^2}{\eta}\frac{\mu_0\overline{\mathbf{j}}\cdot\overline{\mathbf{B}}}{B_{\rm eq}^2}}{1 + \frac{\eta_t}{\eta}\frac{\overline{\mathbf{B}}^2}{B_{\rm eq}^2}}$$

Microscopic feedback

Catastrophic α-quenching ($R_m \gg 1!$) in case of steady state and homogeneous $\overline{\mathbf{B}}$:

$$\alpha = \frac{\alpha_{\rm k}}{1 + R_m\frac{\overline{\mathbf{B}}^2}{B_{\rm eq}^2}}$$

If $\overline{\mathbf{j}}\cdot\overline{\mathbf{B}} \neq 0$ (dynamo generated field) and η_t unquenched:

$$\alpha \approx \eta_t\,\mu_0\frac{\overline{\mathbf{j}}\cdot\overline{\mathbf{B}}}{\overline{\mathbf{B}}^2} \sim \frac{\eta_t}{L} \sim \frac{\eta_t}{l_c}\frac{l_c}{L} \sim \alpha_k\frac{l_c}{L}$$

- In general α-quenching dynamic process: linked to time evolution of helicity
- Boundary conditions matter: Loss of small scale current helicity can alleviate catastrophic quenching
- Catastrophic α-quenching turns large scale dynamo into slow dynamo

3D simulations

Why not just solving the full system to account for all non-linear effects?

- Most systems have $R_e \gg R_m \gg 1$, requiring high resolution
- Large scale dynamos evolve on time scales $\tau_c \ll t \ll \tau_\eta$, requiring long runs compared to convective turn over
- 3D simulations successful for geodynamo
  - $R_m \sim 300$: all relevant magnetic scales resolvable
  - Incompressible system
- Solar dynamo: Ingredients can be simulated
  - Compressible system: density changes by 10^6 through convection zone
  - Boundary layer effects: Tachocline, difficult to simulate (strongly subadiabatic stratification, large time scales)
  - How much resolution required? (CZ about $\sim 10^9$ Mm^3, 1 Mm resolution $\sim 1000^3$ numerical problem)
  - Small scale dynamos can be simulated (for $P_m \sim 1$)

Where did the "first" magnetic field come from?

Meanfield induction equation linear in $\overline{\mathbf{B}}$: possible solution.

$$\frac{\partial\overline{\mathbf{B}}}{\partial t} = \nabla\times\left[\alpha\overline{\mathbf{B}} + (\overline{\mathbf{v}} + \gamma)\times\overline{\mathbf{B}} - (\eta+\eta_t)\,\nabla\times\overline{\mathbf{B}}\right]$$

$\overline{\mathbf{B}} = 0$ is always a valid solution!
Generalized Ohm's law with electron pressure term:

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B} + \frac{1}{\sigma}\mathbf{j} - \frac{1}{\varrho_e}\nabla p_e\,.$$

leads to induction equation with inhomogeneous source term:

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B} - \eta\,\nabla\times\mathbf{B}) + \frac{1}{\varrho_e^2}\nabla\varrho_e\times\nabla p_e\,.$$

Where did the "first" magnetic field come from?

Early universe:

- Ionization fronts from point sources (quasars) driven through an inhomogeneous medium: $1/\varrho_e^2 \nabla \varrho_e \times \nabla p_e$ can lead to about $10^{-23} G$
- Collapse of intergalactic medium to form galaxies leads to 10^{20} G
- Galactic dynamo (growth rate $\sim 3Gy^{-1}$) leads to 10^{-6} G after 10 Gy (today)

Summarizing remarks

Destruction of magnetic field:

- Turbulent diffusivity: cascade of magnetic energy from large scale to dissipation scale (advection+reconnection)
- Enhances dissipation of large field by a factor R_m

Creation of magnetic field:

- Small scale dynamo (non-helical)
  - Amplification of field at and below energy carrying scale of turbulence
  - Stretch-twist-fold-(reconnect)
  - Produces non-helical field and does not require helical motions
  - Controversy: behavior for $P_m \ll 1$
- Large scale dynamo (helical)
  - Amplification of field on scales larger than scale of turbulence
  - Produces helical field and does require helical motions
  - Requires rotation + additional symmetry direction (controversial $\Omega \times J$ effect does not require helical motions)
  - Controversy: catastrophic vs. non-catastrophic quenching