This Week

- Observing Session: Tonight Mar 10 (8:00 pm)

- **MIDTERM**: Thurs Mar 13 (regular class time, 9:30 am)

- Review Session: Wed Mar 12 (5:00 - 7:00 pm)
Today’s Schedule

- Past / Current Homework Questions?
- White Dwarfs and Degeneracy Pressure
- Supernovae and Nuclear Reactions
“Basic” Quantum Mechanics

- Heisenberg Uncertainty Principle
- Pauli Exclusion Principle
- Planck’s Constant:
  \[ h \approx 10^{-34} \text{ J s or } \hbar \equiv h/(2\pi) \text{ J s} \]
Heisenberg Uncertainty Principle

\[ \Delta x \Delta p \geq \frac{\hbar}{2} \]

\[ \Delta t \Delta E \geq \frac{\hbar}{2} \]

- \( p \) is momentum and \( E \) is energy

Werner Heisenberg
Pauli Exclusion Principle

- No two fermions (protons, electrons, neutrons) can occupy the same quantum state

- Fermions have half-integer spin and Bosons (photons) have integer spin

Wolfgang Pauli
Degeneracy Pressure

- White Dwarf: $\sim$ size of Earth, $\sim$ mass of Sun

- Supported by Electron Degeneracy Pressure

$$P_{\text{NR}} = \frac{\hbar^2}{m_e} \left( \frac{Z}{A} \right)^{5/3} \left( \frac{\rho}{m_p} \right)^{5/3}$$
How did you get that result?

Newton:  \( F = ma \)
\( F = \) change in linear momentum per unit time
\( = \frac{\Delta p}{\Delta t} \)
\( p = \) linear momentum
\( = m \times v \)

Before collision:
\(-p_x, p_y, p_z\)

After collision:
\(p_x, p_y, p_z\)

\( \Delta p = 2p_x \)
\( \Delta t = 2 \left( \frac{\Delta x}{v_x} \right) \)
Is there a relationship between Mass and Radius?

\[ P_{\text{HSE}} = P_{\text{Deg}} \Rightarrow R \propto M^{-1/3} \]
Is there a relationship between Mass and Radius?

Yes! How do we find it?
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Use Hydrostatic Equation (who remembers what that even means?)
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Yes! How do we find it?

Use Hydrostatic Equation (who remembers what that even means?)

Set $P_{\text{HSE}} = P_{\text{Deg}} \Rightarrow R \propto M^{-1/3}$
We used $P = nvp$, what happens when $v \approx c$?
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Simply replace $v \rightarrow c$

$$P_R = \hbar c \left( \frac{Z}{A} \right)^{4/3} \left( \frac{\rho}{m_p} \right)^{4/3}$$
Degeneracy Pressure with Numbers

For $Z/A = 1$ and $\rho = 1 \text{ g/cm}^{-3}$

Non-Relativistic:

$$P_{\text{NR}} = 9.9 \times 10^{12} \text{ dyn cm}^{-2}$$

Relativistic:

$$P_{\text{R}} = 1.2 \times 10^{15} \text{ dyn cm}^{-2}$$
Unit conversions are good for the soul, so ...

Convert dyn cm$^{-2}$ (cgs) to SI/MKS unit of pressure: Pascal

Remember $P = F/A$ and a dyn is cgs unit of force
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Convert dyn cm$^{-2}$ (cgs) to SI/MKS unit of pressure: Pascal

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$$1 \text{ dyn cm}^{-1} = 0.1 \text{ Pa}$$
Unit conversions are good for the soul, so ...

Convert \(\text{dyn cm}^{-2}\) (cgs) to SI/MKS unit of pressure: Pascal

Remember \(P = \frac{F}{A}\) and a dyn is cgs unit of force

\[
1 \text{ dyn cm}^{-1} = 0.1 \text{ Pa}
\]

\[
1 \text{ dyn cm}^{-2} = \frac{g}{s^2} \frac{1}{cm^2}
\]

\[
\frac{g}{s^2 \text{ cm}} = \frac{10^{-3} \text{ kg}}{s^2 10^{-2} \text{ m}} = 0.1 \frac{\text{ kg}}{s^2 \text{ m}} = 0.1 \frac{\text{ kg m}}{s^2 \text{ m}^2} = 0.1 \text{ Pa}
\]
Classifying Supernovae – It’s Complicated

**SUPERNOVAE**

- **I**
  - no H lines
  - **Ia**
    - Si lines
    - (Si II, λ=6150 Å)
  - **Ib**
    - He lines
    - (He I, λ=5876 Å)
    - no He lines
  - **Ic**
    - hypernovae

- **II**
  - H lines
  - **II-L**
    - linear
  - **II-P**
    - plateau
    - H recombination
  - **IIpec**
    - peculiar
    - (1987A, III, IV, V)
  - **IIn**
    - narrow emission lines
    - interaction with CSM
  - **IIb**
    - low H

**Population II**
- THERMONUCLEAR EXPLOSION

**Population I**
- CORE-COLLAPSE SUPERNOVAE

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[Image of the diagram]
Supernova Onion Shell Burning

For a 25 solar mass star:

<table>
<thead>
<tr>
<th>Stage</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>H → He</td>
<td>$7 \times 10^6$ years</td>
</tr>
<tr>
<td>He → C</td>
<td>$7 \times 10^5$ years</td>
</tr>
<tr>
<td>C → O</td>
<td>600 years</td>
</tr>
<tr>
<td>O → Si</td>
<td>6 months</td>
</tr>
<tr>
<td>Si → Fe</td>
<td>1 day</td>
</tr>
<tr>
<td>Core Collapse</td>
<td>1/4 second</td>
</tr>
</tbody>
</table>
Why Stop at Iron ($Z = 26$)?

![Diagram showing the relationship between average binding energy per nucleon and number of nucleons in a nucleus. The diagram highlights the nuclear fusion and fission processes.]
Naturally Occurring Elements with $Z > 26$ Exist!

- For high $Z$ elements it is hard to get another charged particle close due to the high Coulomb potential barrier.

- Not for neutrons: $\frac{A}{Z}X + n \rightarrow \frac{A+1}{Z}X + \gamma$

- Results in more massive nuclei that are stable or unstable against beta-decay: $\frac{A+1}{Z}X \rightarrow \frac{A+1}{Z+1}X + ?$
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$\frac{A+1}{Z}X \rightarrow \frac{A+1}{Z+1}X + e^- + \bar{\nu}_e + \gamma$
Neutron Processes

If beta-decay half-life is short compared to timescale for neutron capture

slow process or s-process reactions
tends to produce stable nuclei

If beta-decay half-life is long compared to timescale for neutron capture

rapid process or r-process reactions
tends to produce neutron rich nuclei
Neutron Processes

s-process tend to occur in normal phases of stellar evolution

r-process can occur during a supernova

Neither process plays a significant role in energy production

Accounts for abundances of nuclei with $A \gtrsim 60$, $(Z \gtrsim 26)$