ASTR 1040 Recitation: Stellar Structure

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This Week

- MIDTERM: Thurs Feb 13 (regular class time, 9:30 am)
- Review Session: Wed Feb 12 (5:00 - 7:00 pm)
- Observing Session: Web Feb 12 (7:30 pm)

Today's Schedule

- Comments on Homework
- How to Build a Stellar Structure Model

How To Build A Star

What physics do you need to build a star?

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- Gravity vs. Pressure

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- Gravity vs. Pressure
- Nuclear Reactions
- Energy Transport
- Equation of State

Gravity vs. Pressure

Hydrostatic Balance:

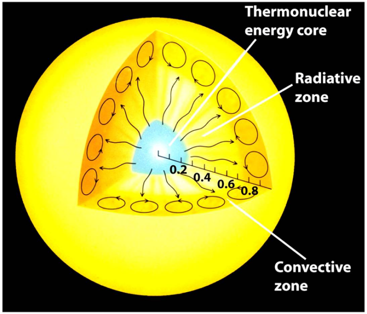
$$\boxed{\frac{dP}{dr} = -\frac{GM_r(r)\rho(r)}{r^2}}$$

How much mass?

$$\boxed{M_r(r) = \int_0^r 4\pi r^2 \rho(r) dr}$$

Gravity vs. Pressure

Q: What is the source of the pressure gradient **outside** the core in the equation for Hydrostatic Equilibrium?



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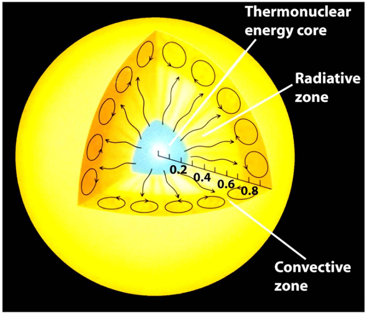

A: Energy transport mechanisms such as radiative diffusion or convection

Radiation exerts a pressure

$$P_{rad} = aT^4/3$$

Nuclear Reactions

Reaction rates, $r_{i,j}$

- $r_{i,j} \approx r_0 X_i X_j \rho^{\alpha+1} T^{\beta}$

Energy released / kg / sec, $\epsilon_{i,j}$

- $\epsilon_{i,j} = \frac{E_0}{\rho} r_{i,j}$ $\quad$ (E_0 = Energy / Rx)

Combine the two equations

- $\epsilon_{i,j} = \epsilon_0' X_i X_j \rho^{\alpha} T^{\beta}$

Nuclear Reactions

Luminosity $\propto$ Energy released

$dL = \epsilon dm$

where $\epsilon = \epsilon_{nuc} + \epsilon_{grav}$ is the *total* energy released / kg / sec by all reactions and gravity

$dm = dM_r = \rho(r)dV = 4\pi r^2 \rho(r)dr$

$$dL_r = \epsilon dM_r = 4\pi r^2 \rho(r)\epsilon dr \Rightarrow \boxed{\frac{dL_r}{dr} = 4\pi r^2 \rho(r)\epsilon}$$

Nuclear Reactions

$$\frac{dL_r}{dr} = 4\pi r^2 \rho(r)\epsilon = 4\pi r^2 \epsilon_0 \rho^{\alpha+1} T^{\beta}$$

Reaction Name	α	β
P-P Chain	1	4
CNO Cycle	1	15
Triple-α	2	40

Energy Transport

Remember radiation exerts a pressure

$$P_{rad} = aT^4/3 \Rightarrow \frac{dP_{rad}}{dr} = \frac{4}{3}aT^3\frac{dT}{dr}$$

From Radiation Transport Theory

$$\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c}F_{rad}$$

Combine equations

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}F_{rad}, \text{ where } F_{rad} = \frac{L_r}{4\pi r^2}$$

Get T in terms of L_r

$$\boxed{\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}}$$

What Equations Do We Have So Far?

1. $\frac{dP}{dr} = -\frac{GM_r(r)\rho(r)}{r^2}$
   - # Eqns = 1, # Variables = 3: $P(r)$, $M_r(r)$, $\rho(r)$

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2. $\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$
   - Eqns = 2, Vars = 3: $P(r)$, $M_r(r)$, $\rho(r)$

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3. $\frac{dL_r}{dr} = 4\pi r^2 \rho(r)\epsilon = 4\pi r^2 \epsilon_0 \rho^{\alpha+1} T^{\beta}$
   - Eqns = 3, Vars = 5: $P(r)$, $M_r(r)$, $\rho(r)$, $L_r(r)$, $T(r)$

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4. $\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$
   - Eqns = 4, Vars = 5: $P(r)$, $M_r(r)$, $\rho(r)$, $L_r(r)$, $T(r)$

Equation of State

Gives the pressure in terms of density and temperature

$$P = P(\rho, T)$$

Ideal Gas:

$$P = \frac{\rho k T}{\bar{m}}, \qquad \bar{m} = \text{mean atomic mass}$$

For the Sun: $\mu = \frac{\bar{m}}{m_H} \approx 1.6$

Or Electron Degenerate Matter:

$$P = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3}$$

Or ...

Final Set of Equaions

1. $\frac{dP}{dr} = -\frac{GM_r(r)\rho(r)}{r^2}$
2. $\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$
3. $\frac{dL_r}{dr} = 4\pi r^2 \rho(r)\epsilon = 4\pi r^2 \epsilon_0 \rho^{\alpha+1} T^{\beta}$
4. $\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$
5. $P = \frac{\rho k T}{\bar{m}}$

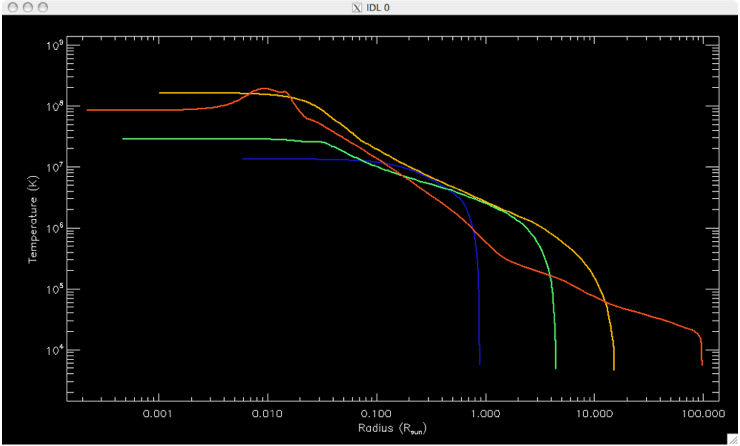
Still cannot solve without Boundary Conditions ...

Boundary Conditions

- $M_r \rightarrow 0$ as $r \rightarrow 0$
- $L_r \rightarrow 0$ as $r \rightarrow 0$
- $\rho \rightarrow 0$ as $r \rightarrow R_*$
- $T \rightarrow T_{eff}$ as $r \rightarrow R_*$
- $P \rightarrow 0$ as $r \rightarrow R_*$

Now we can solve the system

Numerically Integrate the System



Nonstop Pre MS to White Dwarf

Hands-off from pre-main-sequence to white dwarf
Including full helium core flash and AGB pulses.
Run time under 1 hour on Mac Pro using 8 threads.