ASTR 1040 Recitation: Relativity Part III

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March 31 & April 2, 2014

This Week

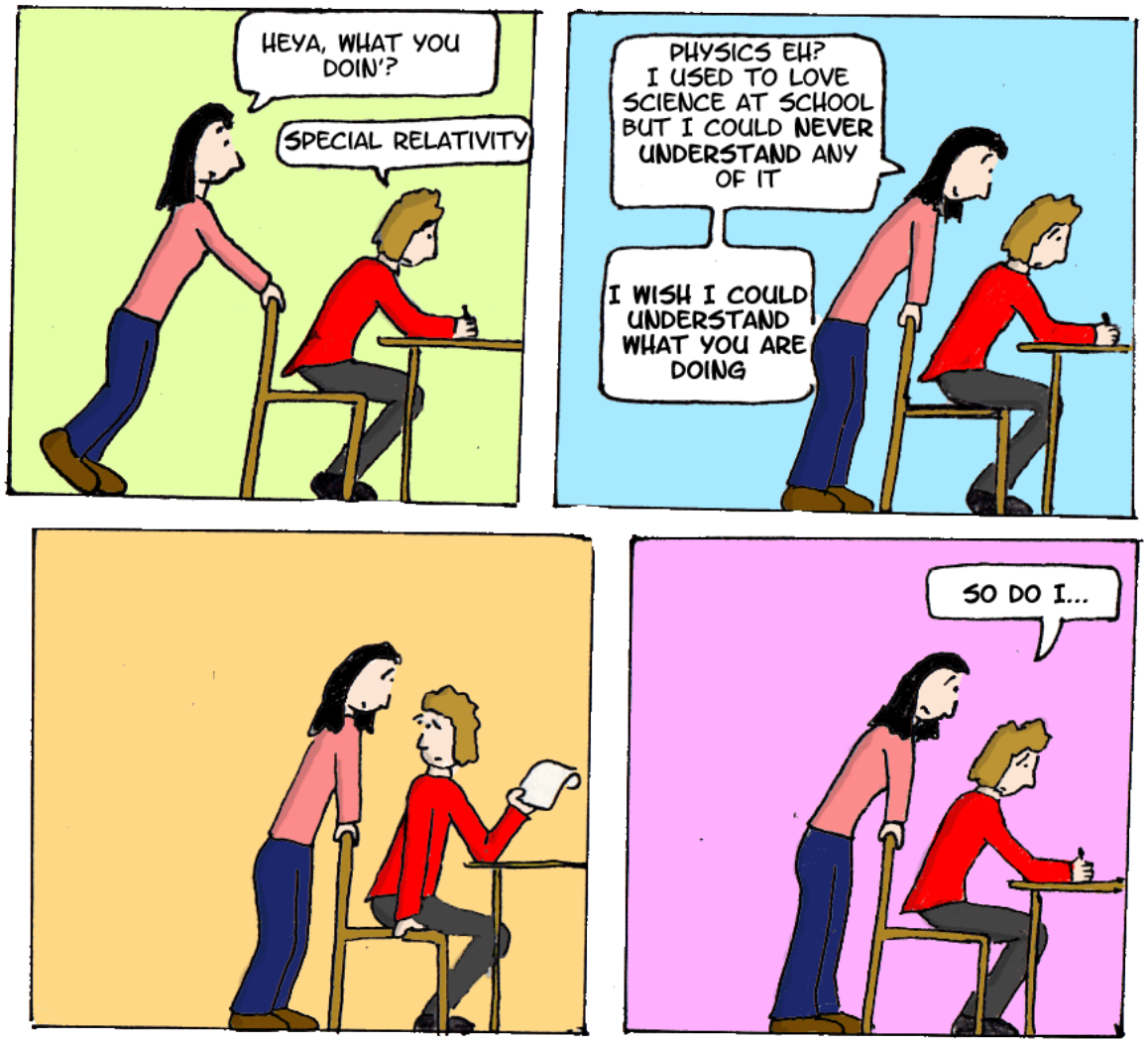
- Night Observing: Wednesday April 2 (8:30 pm)
- Day Observing: Thursday Afternoon
  - Use Heliostat and Hα filters to view the Sun

Today's Schedule

- Past/Current Homework Questions?
- Past/Current Lecture Questions?
- More Special Relativity
- Group Projects

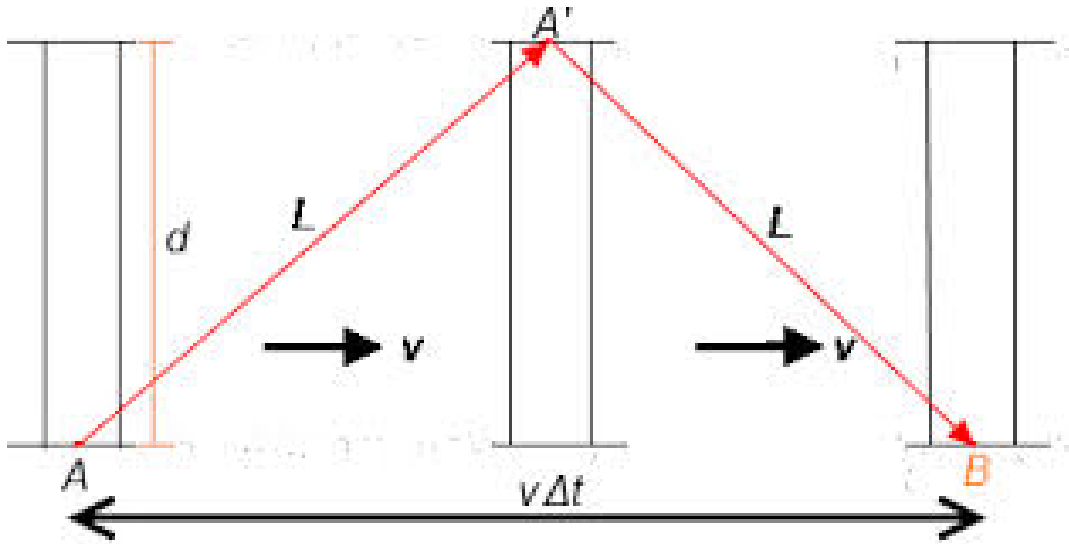
Special Relativity Reminder

- Speed of light is constant for everyone
- Time Dilation
- Length Contraction

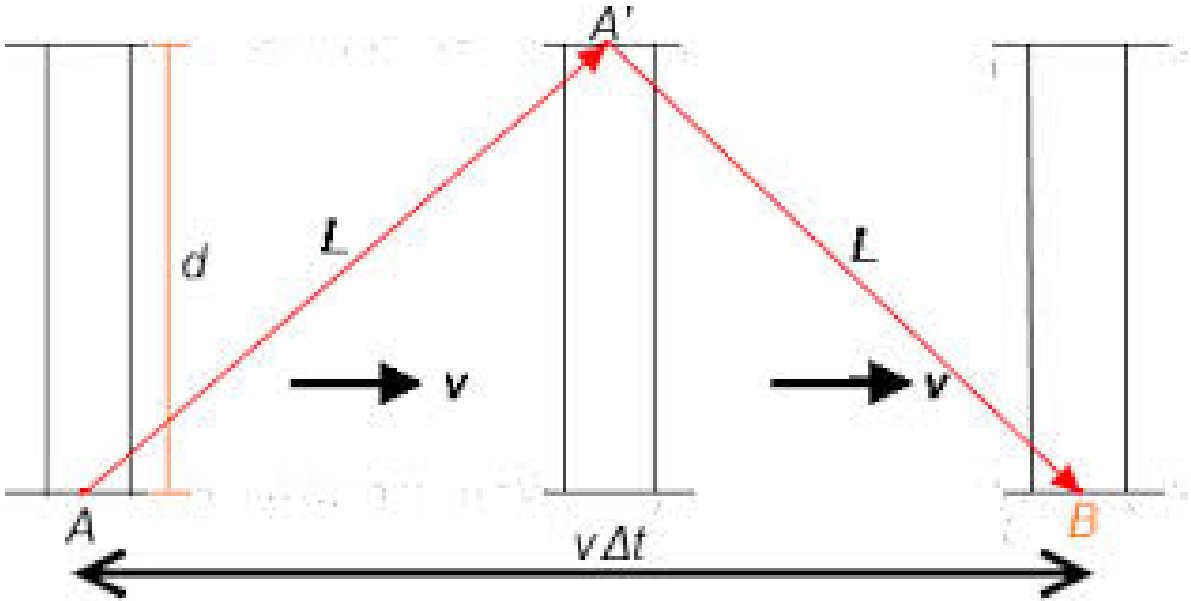


Special Relativity: Time Dilation

- Train car moving to the right at speed v
- Person in train sends laser pulse from ground to a mirror directly above
- What does person on the ground see?



SR Time Dilation



Moving clocks run slow: $t' = \frac{t}{\sqrt{1-(v/c)^2}}$

Lorentz Transformations

Traditional variables

- $ct' = \frac{ct - ux/c}{\sqrt{1-(u/c)^2}}$
- $x' = \frac{x - ut}{\sqrt{1-(u/c)^2}}$
- $y' = y$
- $z' = z$

More compact form

- $ct' = \gamma(ct - \beta x)$
- $x' = \gamma(x - \beta ct)$
- $y' = y$
- $z' = z$

$$\beta \equiv u/c \ \& \ \gamma \equiv (1 - \beta^2)^{-1/2}$$

Group Project I: Velocities

A Train moves with speed u with respect to the ground. What do people on the train measure for your

a) x velocity, v'_x

b) y velocity, v'_y

- $ct' = \gamma(ct - \beta x)$
- $x' = \gamma(x - \beta ct)$
- $y' = y$
- $z' = z$
- $v_x \equiv x/t$
- $v_y \equiv y/t$
- $\beta \equiv u/c$
- $\gamma \equiv (1 - \beta^2)^{-1/2}$

Group Project I: Answer

$$\gamma \equiv \left(1 - \left(\frac{u}{c}\right)^2\right)^{-1/2}$$

a) $$\boxed{v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}}$$

b) $$\boxed{v'_y = \frac{v_y}{\gamma(1 - \frac{uv_x}{c^2})}}$$

Group Project II: Introduction

- Distant galaxy emits radiation in $\hat{y}$ direction in S' frame (the rest frame of the source)
- What angle does the light make with the x axis in the S frame, which moves at speed u with respect to S'?
- $\pi/2$, $> \pi/2$, $< \pi/2$?
- Why?

Group Project II: Introduction

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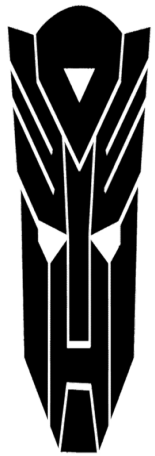
Transformer

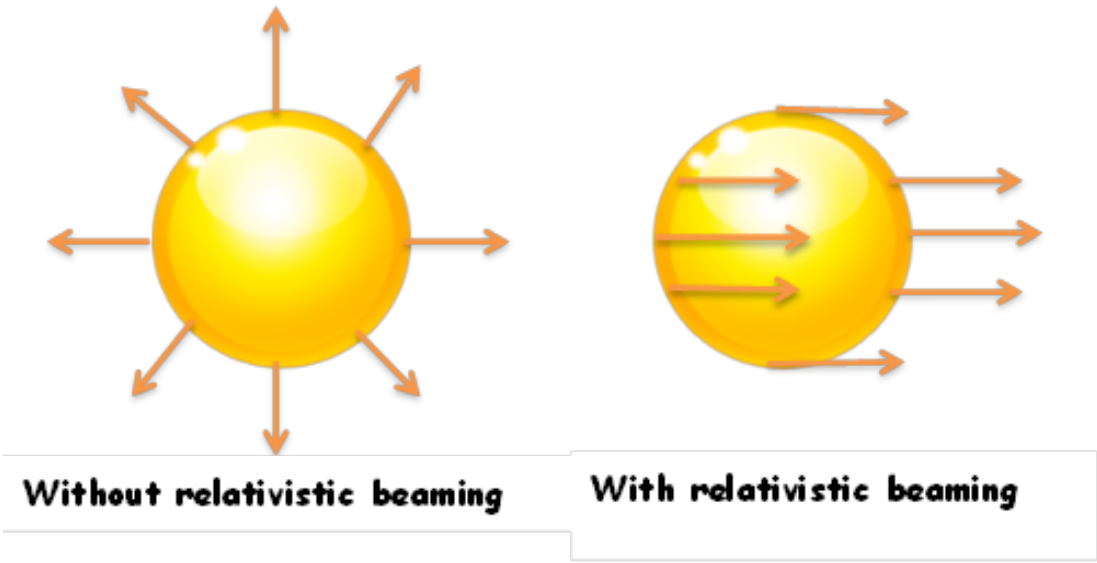


Lorentz Transformer

Group Project II: Doppler Beaming

Galaxy emits light in $\hat{y}$ direction in S' frame. What is $\sin\theta$ in S frame? θ is measured from x axis.

- $v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$
- $v_y = \frac{v'_y}{\gamma(1 + uv'_x/c^2)}$
- $\sin\theta = v_y/v$
- $v = \sqrt{v_x^2 + v_y^2}$



Group Project II: Answer

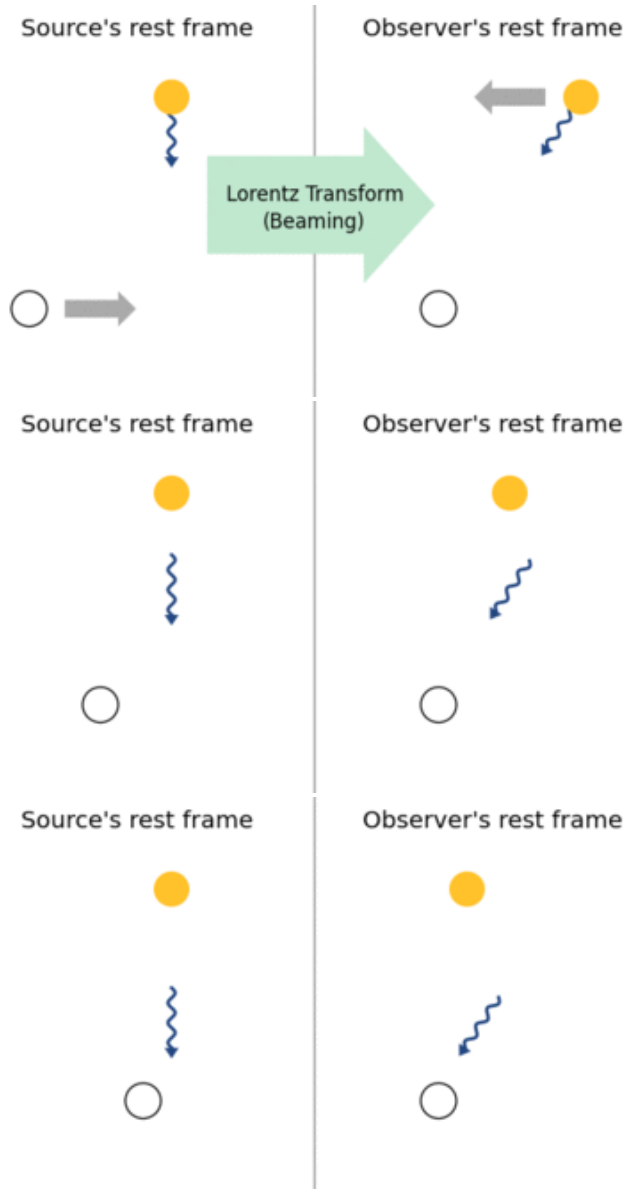
$$\boxed{\sin\theta = \frac{1}{\gamma}}$$

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = u$$

$$v_y = \frac{v'_y}{\gamma(1 + uv'_x/c^2)} = c\sqrt{1 - u^2/c^2}$$

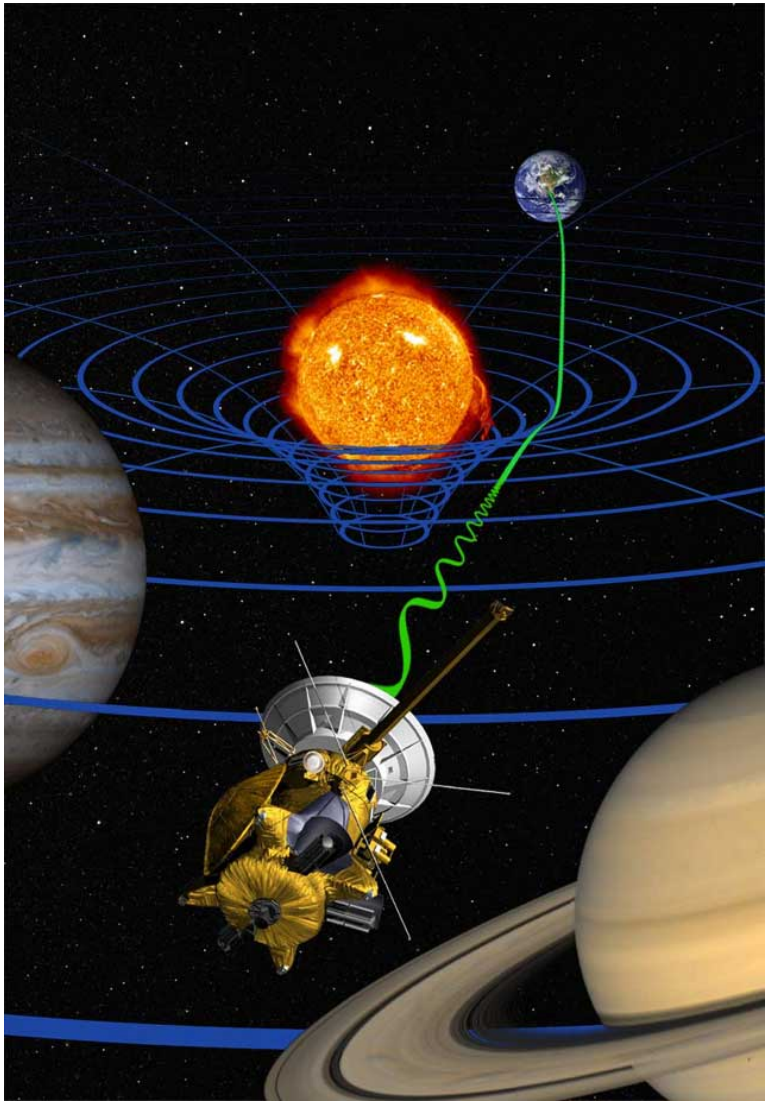
$$v = \sqrt{v_x^2 + v_y^2} = c$$

$$\Rightarrow \sin\theta = v_y/v = \sqrt{1 - u^2/c^2} = 1/\gamma$$



What About General Relativity?

- Special Relativity seems somewhat simple in the kind of math it uses
- What about General Relativity?
- It's just messy



Real General Relativity

- Einstein tensor (Curvature): $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
- Include cosmological constant (Dark Energy): Λ
- Include matter/energy: $T_{\mu\nu}$
- Full Einstein Equations: $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

Real General Relativity

$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ is actually a set of 10 non-linear, partial differential equations, so very very hard to solve