ASTR 1040 Recitation: Relativity Part II

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This Week

- Observing Session: Tues Feb 25 (7:30 pm)

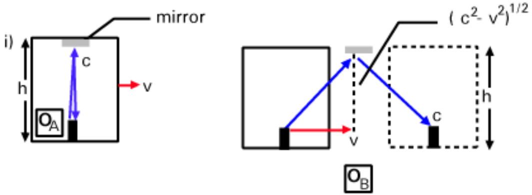
Today's Schedule

- Review a Few Relativity Topics
- Event Horizons – Are They Real??
- Satellite Corrections – Relativity of Everyday Life

Time Dilation

Time Dilation from Special Relativity:

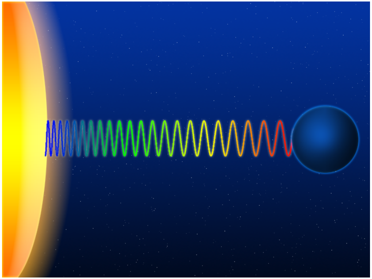
- Moving clocks run slow
- $t = \gamma \tau_p$
- $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$



Time Dilation

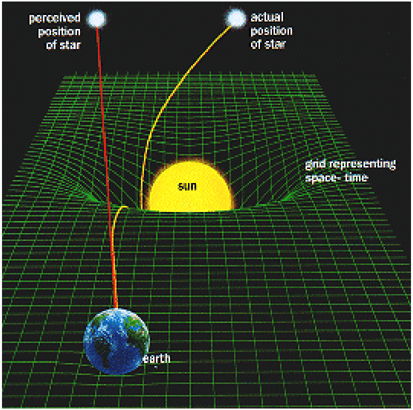
Time Dilation from General Relativity:

- Clocks run slow in gravitational fields
- Light must use a little energy to escape potential well
- Lose energy $\Rightarrow$ lower frequency
- Think of frequency as clock ticks



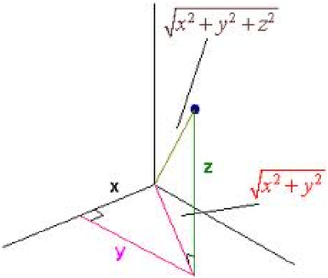
Lensing

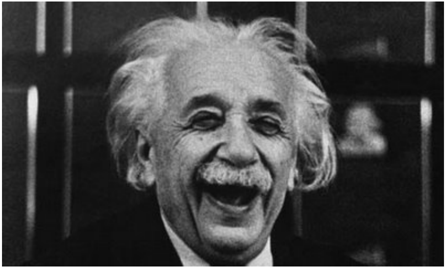
Matter tells space how to curve, curved space-time tells light how to move



Geometry of General Relativity

Geometry you didn't learn in High School





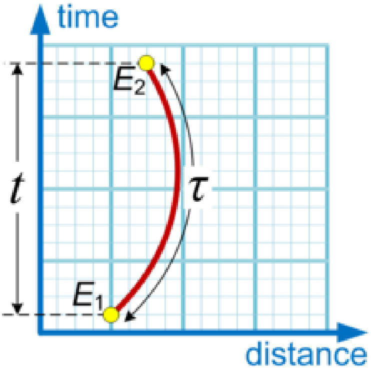
- Constant in any reference frame: $ds^2 = dx^2 + dy^2 + dz^2$

- Constant in any reference frame: $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$ (FLAT Space ONLY)

Proper Time

Proper Time: elapsed time between two events as measured by a clock that passes through both events

- Clock moves through both events
- Move to clock's reference frame
- Events occur at same place, separated in time



Really Hard General Relativity: Metric

Flat Space:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Spherically symmetric matter distribution (Non-rotating, empty space):

$$ds^2 = -B(R)c^2 dt^2 + \frac{dr^2}{B(R)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Proper Time Again

In clock's frame, the events occur at same place

$dr = d\theta = d\phi = 0$ (equivalently: $dx = dy = dz = 0$)

The line elements reduce to:

$$ds^2 = -c^2 dt^2$$

This is a proper time so $dt \to d\tau$

$$\boxed{ds^2 = -c^2 d\tau^2}$$

Event Horizons – Are They Real?

Schwarzschild Black Holes (Non-rotating, empty space):

$B(R) = 1 - \frac{2GM}{c^2 R}$

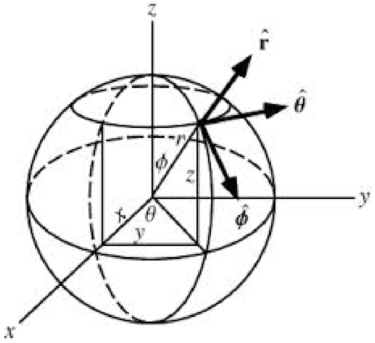
$ds^2 = -\left(1 - \frac{2GM}{c^2 R}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 R}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$

If $B(R) = 0$, the dr coefficient $\rightarrow \infty$

$R_{sch} = \frac{2GM}{c^2}$

Coordinate Singularities

- Compare origin in Polar and Cartesian Coordinates
- Poles of sphere in Spherical Coordinates
- Origin of sphere in Spherical Coordinates



Event Horizons – Are They Real?

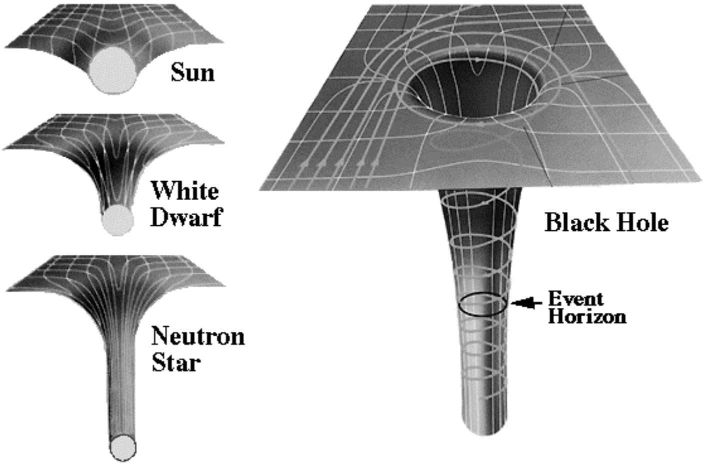
Event horizon is a coordinate singularity

Nothing special happens when you pass through it (not even tidal forces)

What an observer sees as you pass through is a little different

- Remember gravitational time dilation

Weak Gravity



Weak Gravity

Suppose gravitational potential is pretty small: $GM/c^2R \sim \epsilon$

For example: Earth's gravity

How does the line element change?

$$ds^2 = -\left(1 - \frac{2GM}{c^2R}\right) c^2dt^2 + \frac{dr^2}{1-\frac{2GM}{c^2R}} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

Weak Gravity

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Ans: Taylor expand in GM/c^2R

Weak Gravity

Weak gravity line element:

$$ds^2 = -\left(1 - \frac{2GM}{c^2R}\right) c^2dt^2 + (1 + \frac{2GM}{c^2R})dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

Valid for the Earth, Sun, Stars

Not valid for dense objects: Neutron Stars, Black Holes, White Dwarfs (maybe)

Relativistic corrections to satellites

General approach:

- Calculate proper time of satellite in circular orbit with respect to a person at rest at ∞
- Calculate proper time of person on the poles of the Earth (why use the poles and not Boulder?)
- Compare the two results

Satellite Corrections

$$\Phi \equiv -\frac{GM}{R} \qquad \text{and} \qquad \Phi_\oplus \equiv -\frac{GM_\oplus}{c^2 R_\oplus} \approx -21.9 \text{ ms/yr}$$

Proper time of satellite in circular orbit:

Satellite Corrections

$$\Phi \equiv -\frac{GM}{R} \qquad \text{and} \qquad \Phi_\oplus \equiv -\frac{GM_\oplus}{c^2 R_\oplus} \approx -21.9 \text{ ms/yr}$$

Proper time of satellite in circular orbit:

$$\frac{d\tau_{sat}}{dt} = 1 + \frac{\Phi}{c^2} - \frac{v^2}{2c^2}$$

Proper time of person on poles of the Earth:

Satellite Corrections

$$\Phi \equiv -\frac{GM}{R} \qquad \text{and} \qquad \Phi_\oplus \equiv -\frac{GM_\oplus}{c^2 R_\oplus} \approx -21.9 \text{ ms/yr}$$

Proper time of satellite in circular orbit:

$$\frac{d\tau_{sat}}{dt} = 1 + \frac{\Phi}{c^2} - \frac{v^2}{2c^2}$$

Proper time of person on poles of the Earth:

$$\frac{d\tau_{person}}{dt} = 1 + \Phi_\oplus$$

Compare the two:

Satellite Corrections

$$\Phi \equiv -\frac{GM}{R} \qquad \text{and} \qquad \Phi_\oplus \equiv -\frac{GM_\oplus}{c^2 R_\oplus} \approx -21.9 \text{ ms/yr}$$

Proper time of satellite in circular orbit:

$$\frac{d\tau_{sat}}{dt} = 1 + \frac{\Phi}{c^2} - \frac{v^2}{2c^2}$$

Proper time of person on poles of the Earth:

$$\frac{d\tau_{person}}{dt} = 1 + \Phi_\oplus$$

Compare the two:

$$\frac{d\tau_{sat}}{dt} - \frac{d\tau_{person}}{dt} = \frac{\Phi}{c^2} - \Phi_\oplus - \frac{v^2}{2c^2}$$

Satellite Corrections

$$\frac{d\tau_{sat}}{dt} - \frac{d\tau_{person}}{dt} = \frac{\Phi}{c^2} - \Phi_\oplus - \frac{v^2}{2c^2}$$

$$\frac{d\tau_{sat}}{dt} - \frac{d\tau_{person}}{dt} = -\Phi_\oplus \left(-\frac{R_\oplus}{2R} + 1 - \frac{R_\oplus}{R}\right)$$

$$\frac{d\tau_{sat}}{dt} - \frac{d\tau_{person}}{dt} = -\Phi_\oplus (C_{SR} + C_{GR}) = f_{SR} + f_{GR}$$

Real numbers:

- ISS: $R \sim 6800$ km, $v \sim 7.66$ km/s
- $f_{SR} \sim -10.3$ ms/yr, $f_{GR} \sim 1.35$ ms/yr $\Rightarrow -8.95$ ms/yr

- GPS: $R \sim 2.66 \times 10^7$ m, $v \sim 3.89$ km/s
- $f_{SR} \sim -2.65$ ms/yr, $f_{GR} \sim 16.7$ ms/yr $\Rightarrow +14.05$ ms/yr